8.3 A plane wave traveling in a medium with $\varepsilon_{r_1} = 9$ is normally incident upon a second medium with $\varepsilon_{r_2} = 4$. Both media are made of nonmagnetic, non-conducting materials. If the magnetic field of the incident plane wave is given by

$$\mathbf{H}^i = \hat{\mathbf{z}} 2\cos(2\pi \times 10^9 t - ky) \qquad \text{(A/m)}.$$

(a) Obtain time-domain expressions for the electric and magnetic fields in each of the two media.

(b) Determine the average power densities of the incident, reflected, and transmitted waves.

Solution:

(a) In medium 1,

$$u_p = \frac{c}{\sqrt{\varepsilon_{r_1}}} = \frac{3 \times 10^8}{\sqrt{9}} = 1 \times 10^8 \qquad \text{(m/s)},$$

$$k_1 = \frac{\omega}{u_p} = \frac{2\pi \times 10^9}{1 \times 10^8} = 20\pi \qquad \text{(rad/m)},$$

$$\mathbf{H}^i = \hat{\mathbf{z}} 2\cos(2\pi \times 10^9 t - 20\pi y) \qquad \text{(A/m)},$$

$$\eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_{r_1}}} = \frac{377}{3} = 125.67\ \Omega,$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{r_2}}} = \frac{377}{2} = 188.5\ \Omega,$$

$$\begin{aligned}\mathbf{E}^i &= -\hat{\mathbf{x}} 2\eta_1 \cos(2\pi \times 10^9 t - 20\pi y) \\ &= -\hat{\mathbf{x}} 251.34\cos(2\pi \times 10^9 t - 20\pi y) \qquad \text{(V/m)},\end{aligned}$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{188.5 - 125.67}{188.5 + 125.67} = 0.2,$$

$$\tau = 1 + \Gamma = 1.2,$$

$$\begin{aligned}\mathbf{E}^r &= -\hat{\mathbf{x}} 251.34 \times 0.2\cos(2\pi \times 10^9 t + 20\pi y) \\ &= -\hat{\mathbf{x}} 50.27\cos(2\pi \times 10^9 t + 20\pi y) \qquad \text{(V/m)},\end{aligned}$$

$$\begin{aligned}\mathbf{H}^r &= -\hat{\mathbf{z}} \frac{50.27}{\eta_1} \cos(2\pi \times 10^9 t + 20\pi y) \\ &= -\hat{\mathbf{z}} 0.4\cos(2\pi \times 10^9 t + 20\pi y) \qquad \text{(A/m)},\end{aligned}$$

$$\mathbf{E}_1 = \mathbf{E}^i + \mathbf{E}^r$$

$$= -\hat{\mathbf{x}}\,[25.134\cos(2\pi\times 10^9 t - 20\pi y) + 50.27\cos(2\pi\times 10^9 t + 20\pi y)] \qquad \text{(V/m)},$$

$$\mathbf{H}_1 = \mathbf{H}^{\mathrm{i}} + \mathbf{H}^{\mathrm{r}} = \hat{\mathbf{z}}\,[2\cos(2\pi\times 10^9 t - 20\pi y) - 0.4\cos(2\pi\times 10^9 t + 20\pi y)] \qquad \text{(A/m)}.$$

In medium 2,

$$k_2 = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\,k_1 = \sqrt{\frac{4}{9}}\times 20\pi = \frac{40\pi}{3} \qquad \text{(rad/m)},$$

$$\mathbf{E}_2 = \mathbf{E}^{\mathrm{t}} = -\hat{\mathbf{x}}\,251.34\,\tau\cos\left(2\pi\times 10^9 t - \frac{40\pi y}{3}\right)$$

$$= -\hat{\mathbf{x}}\,301.61\cos\left(2\pi\times 10^9 t - \frac{40\pi y}{3}\right) \qquad \text{(V/m)},$$

$$\mathbf{H}_2 = \mathbf{H}^{\mathrm{t}} = \hat{\mathbf{z}}\,\frac{301.61}{\eta_2}\cos\left(2\pi\times 10^9 t - \frac{40\pi y}{3}\right)$$

$$= \hat{\mathbf{z}}\,1.6\cos\left(2\pi\times 10^9 t - \frac{40\pi y}{3}\right) \qquad \text{(A/m)}.$$

(b)

$$\mathbf{S}^{\mathrm{i}}_{\mathrm{av}} = \hat{\mathbf{y}}\,\frac{|E_0|^2}{2\eta_1} = \hat{\mathbf{y}}\,\frac{(251.34)^2}{2\times 125.67} = \hat{\mathbf{y}}\,251.34 \qquad \text{(W/m}^2\text{)},$$

$$\mathbf{S}^{\mathrm{r}}_{\mathrm{av}} = -\hat{\mathbf{y}}\,|\Gamma|^2(251.34) = \hat{\mathbf{y}}\,10.05 \qquad \text{(W/m}^2\text{)},$$

$$\mathbf{S}^{\mathrm{t}}_{\mathrm{av}} = \hat{\mathbf{y}}\,(251.34 - 10.05) = \hat{\mathbf{y}}\,241.29 \qquad \text{(W/m}^2\text{)}.$$