

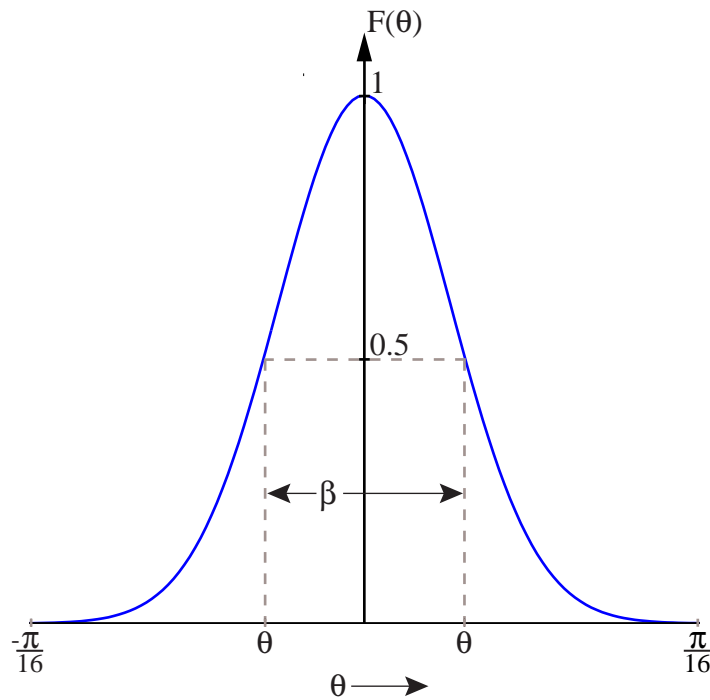
**9.12** The normalized radiation intensity of a certain antenna is given by

$$F(\theta) = \exp(-20\theta^2) \quad \text{for } 0 \leq \theta \leq \pi$$

where  $\theta$  is in radians. Determine:

- (a) The half-power beamwidth.
- (b) The pattern solid angle.
- (c) The antenna directivity.

**Solution:**



**Figure P9.12**  $F(\theta)$  versus  $\theta$ .

(a) Since  $F(\theta)$  is independent of  $\phi$ , the beam is symmetrical about  $z = 0$ . Upon setting  $F(\theta) = 0.5$ , we have

$$F(\theta) = \exp(-20\theta^2) = 0.5,$$

$$\ln[\exp(-20\theta^2)] = \ln(0.5),$$

$$20\theta^2 = -0.693,$$

$$\theta = \pm \left( \frac{0.693}{20} \right)^{1/2} = \pm 0.186 \text{ radians.}$$

Hence,  $\beta = 2 \times 0.186 = 0.372 \text{ radians} = 21.31^\circ$ .

(b) By Eq. (9.21),

$$\begin{aligned} \Omega_p &= \iint_{4\pi} F(\theta) \sin \theta \, d\theta \, d\phi \\ &= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \exp(-20\theta^2) \sin \theta \, d\theta \, d\phi \\ &= 2\pi \int_0^{\pi} \exp(-20\theta^2) \sin \theta \, d\theta. \end{aligned}$$

Numerical evaluation yields

$$\Omega_p = 0.156 \text{ sr.}$$

(c)

$$D = \frac{4\pi}{\Omega_p} = \frac{4\pi}{0.156} = 80.55.$$


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