9.45 A three-element linear array of isotropic sources aligned along the z axis has an interelement spacing of $\lambda/4$ (Fig. P9.45). The amplitude excitation of the center element is twice that of the bottom and top elements, and the phases are $-\pi/2$ for the bottom element and $\pi/2$ for the top element, relative to that of the center element. Determine the array factor and plot it in the elevation plane.

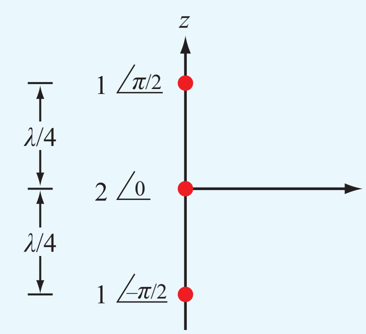

Figure P9.45 Three-element array of Problem 9.45.

Solution: From Eq. (9.110),

$$
\begin{aligned}
F_{\mathrm{a}}(\theta) &= \left|\sum_{i=0}^{2} a_i \exp j\psi_i \exp jikd\cos\theta\right|^2 \\
&= \left|a_0 \exp j\psi_0 + a_1 \exp j\psi_1 \exp jkd\cos\theta + a_2 \exp j\psi_2 \exp j2kd\cos\theta\right|^2 \\
&= \big|\exp j(\psi_1 - \pi/2) + 2\exp j\psi_1 \exp j(2\pi/\lambda)(\lambda/4)\cos\theta \\
&\qquad + \exp j(\psi_1 + \pi/2)\exp j2(2\pi/\lambda)(\lambda/4)\cos\theta\big|^2 \\
&= \left|\exp j\psi_1 \exp j(\pi/2)\cos\theta\right|^2 \big|\exp -j\pi/2 \exp -j(\pi/2)\cos\theta \\
&\qquad + 2 + \exp j\pi/2 \exp j(\pi/2)\cos\theta\big|^2 \\
&= 4\left(1+\cos\left(\tfrac{1}{2}\pi(1+\cos\theta)\right)\right)^2, \\
F_{\mathrm{an}}(\theta) &= \tfrac{1}{4}\left(1+\cos\left(\tfrac{1}{2}\pi(1+\cos\theta)\right)\right)^2.
\end{aligned}
$$

This normalized array factor is shown in Fig. 9.45(a).

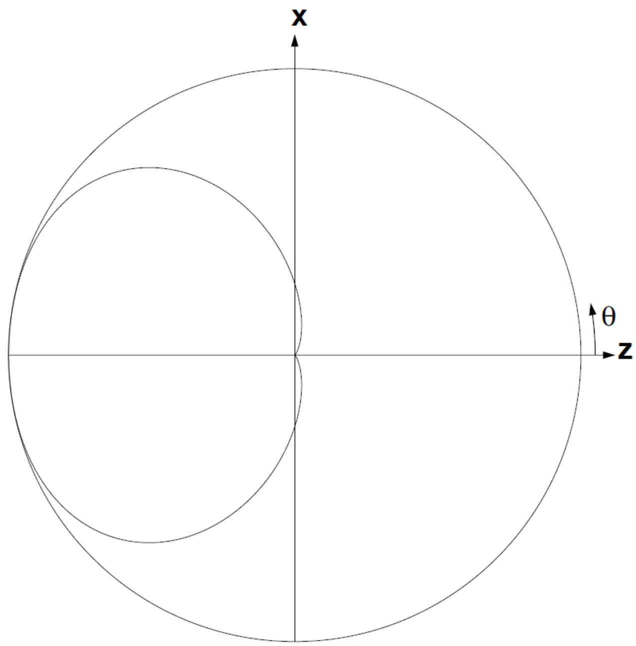

Figure P9.45: (a) Normalized array pattern of the 3-element array of Problem 9.45.