1.19 Complex numbers z_1 and z_2 are given by

$$z_1 = -3 + j2$$
$$z_2 = 1 - j2$$

Determine **(a)** $z_1 z_2$, **(b)** z_1/z_2^*, **(c)** z_1^2, and **(d)** $z_1 z_1^*$, all in polar form.

Solution:

(a) We first convert z_1 and z_2 to polar form:

$$\begin{aligned} z_1 = -(3 - j2) &= -\left(\sqrt{3^2 + 2^2}\, e^{-j\tan^{-1} 2/3}\right) \\ &= -\sqrt{13}\, e^{-j33.7^\circ} \\ &= \sqrt{13}\, e^{j(180^\circ - 33.7^\circ)} \\ &= \sqrt{13}\, e^{j146.3^\circ}. \end{aligned}$$

$$\begin{aligned} z_2 = 1 - j2 &= \sqrt{1+4}\, e^{-j\tan^{-1} 2} \\ &= \sqrt{5}\, e^{-j63.4^\circ}. \end{aligned}$$

$$\begin{aligned} z_1 z_2 &= \sqrt{13}\, e^{j146.3^\circ} \times \sqrt{5}\, e^{-j63.4^\circ} \\ &= \sqrt{65}\, e^{j82.9^\circ}. \end{aligned}$$

(b)

$$\frac{z_1}{z_2^*} = \frac{\sqrt{13}\, e^{j146.3^\circ}}{\sqrt{5}\, e^{j63.4^\circ}} = \sqrt{\frac{13}{5}}\, e^{j82.9^\circ}.$$

(c)

$$\begin{aligned} z_1^2 = (\sqrt{13})^2 (e^{j146.3^\circ})^2 &= 13 e^{j292.6^\circ} \\ &= 13 e^{-j360^\circ} e^{j292.6^\circ} \\ &= 13 e^{-j67.4^\circ}. \end{aligned}$$

(c)

$$\begin{aligned} z_1 z_1^* &= \sqrt{13}\, e^{j146.3^\circ} \times \sqrt{13}\, e^{-j146.3^\circ} \\ &= 13. \end{aligned}$$