1.20 If $z = -2 + j4$, determine the following quantities in polar form:

(a) $1/z$,
(b) z^3,
(c) $|z|^2$,
(d) $\mathfrak{Im}\{z\}$,
(e) $\mathfrak{Im}\{z^*\}$.

Solution: (Note: In the following solutions, numbers are expressed to only two decimal places, but the final answers are found using a calculator with 10 decimal places.)

(a)

$$\frac{1}{z} = \frac{1}{-2+j4} = (-2+j4)^{-1} = (4.47\,e^{j116.6^\circ})^{-1} = (4.47)^{-1}e^{-j116.6^\circ} = 0.22\,e^{-j116.6^\circ}.$$

(b) $z^3 = (-2+j4)^3 = (4.47\,e^{j116.6^\circ})^3 = (4.47)^3 e^{j350.0^\circ} = 89.44\,e^{-j10^\circ}$.
(c) $|z|^2 = z \cdot z^* = (-2+j4)(-2-j4) = 4+16 = 20$.
(d) $\mathfrak{Im}\{z\} = \mathfrak{Im}\{-2+j4\} = 4$.
(e) $\mathfrak{Im}\{z^*\} = \mathfrak{Im}\{-2-j4\} = -4 = 4e^{j\pi}$.