1.26 A voltage source given by

$$\upsilon_s(t) = 25\cos(2\pi \times 10^3 t - 30^\circ) \qquad \text{(V)}$$

is connected to a series RC load as shown in Fig. 1-20. If $R = 1$ MΩ and $C = 200$ pF, obtain an expression for $\upsilon_c(t)$, the voltage across the capacitor.

Solution: In the phasor domain, the circuit is a voltage divider, and

$$\widetilde{V}_c = \widetilde{V}_s \frac{1/j\omega C}{R + 1/j\omega C} = \frac{\widetilde{V}_s}{(1 + j\omega RC)}.$$

Now $\widetilde{V}_s = 25 \exp -j30^\circ$ V with $\omega = 2\pi \times 10^3$ rad/s, so

$$\begin{aligned}\widetilde{V}_c &= \frac{25\exp -j30^\circ \text{ V}}{1 + j((2\pi \times 10^3 \text{ rad/s}) \times (10^6\ \Omega) \times (200 \times 10^{-12} \text{ F}))} \\ &= \frac{25\exp -j30^\circ \text{ V}}{1 + j2\pi/5} = 15.57\exp -j81.5^\circ \text{ V}.\end{aligned}$$

Converting back to an instantaneous value,

$$\begin{aligned}v_c(t) = \mathfrak{Re}\widetilde{V}_c \exp j\omega t &= \mathfrak{Re}15.57\exp j(\omega t - 81.5^\circ) \text{ V} \\ &= 15.57\cos\left(2\pi \times 10^3 t - 81.5^\circ\right) \text{ V},\end{aligned}$$

where t is expressed in seconds.