

1.3 A 2 kHz sound wave traveling in the x -direction in air was observed to have a differential pressure $p(x, t) = 10 \text{ N/m}^2$ at $x = 0$ and $t = 50 \mu\text{s}$. If the reference phase of $p(x, t)$ is 36° , find a complete expression for $p(x, t)$. The velocity of sound in air is 330 m/s.

Solution: The general form is given by Eq. (1.17),

$$p(x, t) = A \cos \left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0 \right),$$

where it is given that $\phi_0 = 36^\circ$. From Eq. (1.26), $T = 1/f = 1/(2 \times 10^3) = 0.5 \text{ ms}$. From Eq. (1.27),

$$\lambda = \frac{u_p}{f} = \frac{330}{2 \times 10^3} = 0.165 \text{ m}.$$

Also, since

$$\begin{aligned} p(x = 0, t = 50 \mu\text{s}) &= 10 \text{ (N/m}^2\text{)} = A \cos \left(\frac{2\pi \times 50 \times 10^{-6}}{5 \times 10^{-4}} + 36^\circ \frac{\pi \text{ rad}}{180^\circ} \right) \\ &= A \cos(1.26 \text{ rad}) = 0.31A, \end{aligned}$$

it follows that $A = 10/0.31 = 32.36 \text{ N/m}^2$. So, with t in (s) and x in (m),

$$\begin{aligned} p(x, t) &= 32.36 \cos \left(2\pi \times 10^6 \frac{t}{500} - 2\pi \times 10^3 \frac{x}{165} + 36^\circ \right) \quad (\text{N/m}^2) \\ &= 32.36 \cos(4\pi \times 10^3 t - 12.12\pi x + 36^\circ) \quad (\text{N/m}^2). \end{aligned}$$
