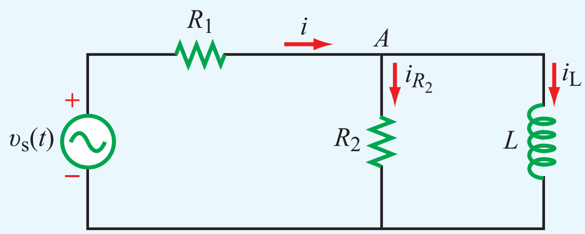
1.30 The voltage source of the circuit shown in Fig. P1.30 is given by

$$\upsilon_s(t) = 25\cos(4\times 10^4 t - 45^\circ) \qquad \text{(V)}.$$

Obtain an expression for $i_L(t)$, the current flowing through the inductor.



$R_1 = 20\ \Omega$, $R_2 = 30\ \Omega$, $L = 0.4$ mH

Figure P1.30 Circuit for Problem 1.30.

Solution: Based on the given voltage expression, the phasor source voltage is

$$\widetilde{V}_s = 25e^{-j45^\circ} \qquad \text{(V)}. \tag{1}$$

The voltage equation for the left-hand side loop is

$$R_1 i + R_2 i_{R_2} = \upsilon_s \tag{2}$$

For the right-hand loop,

$$R_2 i_{R_2} = L\frac{di_L}{dt}\,, \tag{3}$$

and at node A,

$$i = i_{R_2} + i_L. \tag{4}$$

Next, we convert Eqs. (2)–(4) into phasor form:

$$R_1\tilde{I} + R_2\tilde{I}_{R_2} = \widetilde{V}_s \tag{5}$$

$$R_2\tilde{I}_{R_2} = j\omega L\tilde{I}_L \tag{6}$$

$$\tilde{I} = \tilde{I}_{R_2} + \tilde{I}_L \tag{7}$$

Upon combining (6) and (7) to solve for $\tilde{I}_{R_2}$ in terms of $\tilde{I}$, we have:

$$\tilde{I}_{R_2} = \frac{j\omega L}{R_2 + j\omega L} I\,. \tag{8}$$

Substituting (8) in (5) and then solving for $\tilde{I}$ leads to:

$$R_1\tilde{I} + \frac{jR_2\omega L}{R_2 + j\omega L}\tilde{I} = \widetilde{V}_s$$

$$\begin{aligned}
\tilde{I}\left(R_1+\frac{jR_2\omega L}{R_2+j\omega L}\right)&=\widetilde{V}_{\rm s}\\
\tilde{I}\left(\frac{R_1R_2+jR_1\omega L+jR_2\omega L}{R_2+j\omega L}\right)&=\widetilde{V}_{\rm s}\\
\tilde{I}=\left(\frac{R_2+j\omega L}{R_1R_2+j\omega L(R_1+R_2)}\right)&\widetilde{V}_{\rm s}.
\end{aligned}\tag{9}$$

Combining (6) and (7) to solve for $\tilde{I}_{\rm L}$ in terms of $\tilde{I}$ gives

$$\tilde{I}_{\rm L}=\frac{R_2}{R_2+j\omega L}\tilde{I}.\tag{10}$$

Combining (9) and (10) leads to

$$\begin{aligned}
\tilde{I}_{\rm L}&=\left(\frac{R_2}{R_2+j\omega L}\right)\left(\frac{R_2+j\omega L}{R_1R_2+j\omega L(R_1+R_2)}\right)\widetilde{V}_{\rm s}\\
&=\frac{R_2}{R_1R_2++j\omega L(R_1+R_2)}\widetilde{V}_{\rm s}.
\end{aligned}$$

Using (1) for $\widetilde{V}_{\rm s}$ and replacing R_1, R_2, L and ω with their numerical values, we have

$$\begin{aligned}
\tilde{I}_{\rm L}&=\frac{30}{20\times 30+j4\times 10^4\times 0.4\times 10^{-3}(20+30)}\,25e^{-j45^\circ}\\
&=\frac{30\times 25}{600+j800}\,e^{-j45^\circ}\\
&=\frac{7.5}{6+j8}\,e^{-j45^\circ}=\frac{7.5e^{-j45^\circ}}{10e^{j53.1^\circ}}=0.75e^{-j98.1^\circ}\qquad\text{(A)}.
\end{aligned}$$

Finally,

$$\begin{aligned}
i_{\rm L}(t)&=\mathfrak{Re}[\tilde{I}_{\rm L}e^{j\omega t}]\\
&=0.75\cos(4\times 10^4 t-98.1^\circ)\qquad\text{(A)}.
\end{aligned}$$