10.4 A geostationary satellite is at a distance of 40,000 km from a ground receiving station. The satellite transmitting antenna is a circular aperture with a 1-m diameter, and the ground station uses a parabolic dish antenna with an effective diameter of 20 cm. If the satellite transmits 1 kW of power at 12 GHz and the ground receiver is characterized by a system noise temperature of 1,000 K, what would be the signal-to-noise ratio of a received TV signal with a bandwidth of 6 MHz? The antennas and the atmosphere may be assumed lossless.

Solution: We are given

$$R = 4 \times 10^7 \text{ m}, \qquad d_\mathrm{t} = 1 \text{ m}, \qquad d_\mathrm{r} = 0.2 \text{ m}, \qquad P_\mathrm{t} = 10^3 \text{ W},$$
$$f = 12 \text{ GHz}, \qquad T_\mathrm{sys} = 1{,}000 \text{ K}, \qquad B = 6 \text{ MHz}.$$

At $f = 12$ GHz, $\lambda = c/f = 3 \times 10^8/12 \times 10^9 = 2.5 \times 10^{-2}$ m. With $\xi_\mathrm{t} = \xi_\mathrm{r} = 1$,

$$G_\mathrm{t} = D_\mathrm{t} = \frac{4\pi A_\mathrm{t}}{\lambda^2} = \frac{4\pi(\pi d_\mathrm{t}^2/4)}{\lambda^2} = \frac{4\pi \times \pi \times 1}{4 \times (2.5 \times 10^{-2})^2} = 15{,}791.37,$$
$$G_\mathrm{r} = D_\mathrm{r} = \frac{4\pi A_r}{\lambda^2} = \frac{4\pi(\pi d_\mathrm{r}^2/4)}{\lambda^2} = \frac{4\pi \times \pi(0.2)^2}{4 \times (2.5 \times 10^{-2})^2} = 631.65.$$

Applying Eq. (10.11) with $\Upsilon(\theta) = 1$ gives:

$$S_\mathrm{n} = \frac{P_\mathrm{t} G_\mathrm{t} G_\mathrm{r}}{K T_\mathrm{sys} B} \left(\frac{\lambda}{4\pi R}\right)^2 = \frac{10^3 \times 15{,}791.37 \times 631.65}{1.38 \times 10^{-23} \times 10^3 \times 6 \times 10^6} \left(\frac{2.5 \times 10^{-2}}{4\pi \times 4 \times 10^7}\right)^2 = 298.$$