2.13 In addition to not dissipating power, a lossless line has two important features: (1) it is dispersionless (u_p is independent of frequency); and (2) its characteristic impedance Z_0 is purely real. Sometimes, it is not possible to design a transmission line such that $R' \ll \omega L'$ and $G' \ll \omega C'$, but it is possible to choose the dimensions of the line and its material properties so as to satisfy the condition

$$R'C' = L'G' \qquad \text{(distortionless line)}$$

Such a line is called a ***distortionless*** line, because despite the fact that it is not lossless, it nonetheless possesses the previously mentioned features of the lossless line. Show that for a distortionless line,

$$\alpha = R'\sqrt{\frac{C'}{L'}} = \sqrt{R'G'}\,,$$
$$\beta = \omega\sqrt{L'C'}\,,$$
$$Z_0 = \sqrt{\frac{L'}{C'}}\,.$$

Solution: Using the distortionless condition in Eq. (2.22) gives

$$\begin{aligned}
\gamma = \alpha + j\beta &= \sqrt{(R' + j\omega L')(G' + j\omega C')} \\
&= \sqrt{L'C'}\sqrt{\left(\frac{R'}{L'} + j\omega\right)\left(\frac{G'}{C'} + j\omega\right)} \\
&= \sqrt{L'C'}\sqrt{\left(\frac{R'}{L'} + j\omega\right)\left(\frac{R'}{L'} + j\omega\right)} \\
&= \sqrt{L'C'}\left(\frac{R'}{L'} + j\omega\right) = R'\sqrt{\frac{C'}{L'}} + j\omega\sqrt{L'C'}.
\end{aligned}$$

Hence,

$$\alpha = \mathfrak{Re}(\gamma) = R'\sqrt{\frac{C'}{L'}}\,, \qquad \beta = \mathfrak{Im}(\gamma) = \omega\sqrt{L'C'}\,, \qquad u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}}\,.$$

Similarly, using the distortionless condition in Eq. (2.29) gives

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \sqrt{\frac{L'}{C'}}\sqrt{\frac{R'/L' + j\omega}{G'/C' + j\omega}} = \sqrt{\frac{L'}{C'}}\,.$$