

**2.19** A  $50\ \Omega$  lossless transmission line is terminated in a load with impedance  $Z_L = (30 - j50)\ \Omega$ . The wavelength is 8 cm. Determine:

- (a) The reflection coefficient at the load.
- (b) The standing-wave ratio on the line.
- (c) The position of the voltage maximum nearest the load.
- (d) The position of the current maximum nearest the load.
- (e) Verify quantities in parts (a)–(d) using Module 2.4. Include a printout of the screen display.

**Solution:**

- (a) From Eq. (2.59),

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(30 - j50) - 50}{(30 - j50) + 50} = 0.57 \exp -j79.8^\circ.$$

- (b) From Eq. (2.73),

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.57}{1 - 0.57} = 3.65.$$

- (c) From Eq. (2.70)

$$\begin{aligned} d_{\max} &= \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2} = \frac{-79.8^\circ \times 8\text{ cm}}{4\pi} \frac{\pi\text{ rad}}{180^\circ} + \frac{n \times 8\text{ cm}}{2} \\ &= -0.89\text{ cm} + 4.0\text{ cm} = 3.11\text{ cm}. \end{aligned}$$

- (d) A current maximum occurs at a voltage minimum, and from Eq. (2.72),

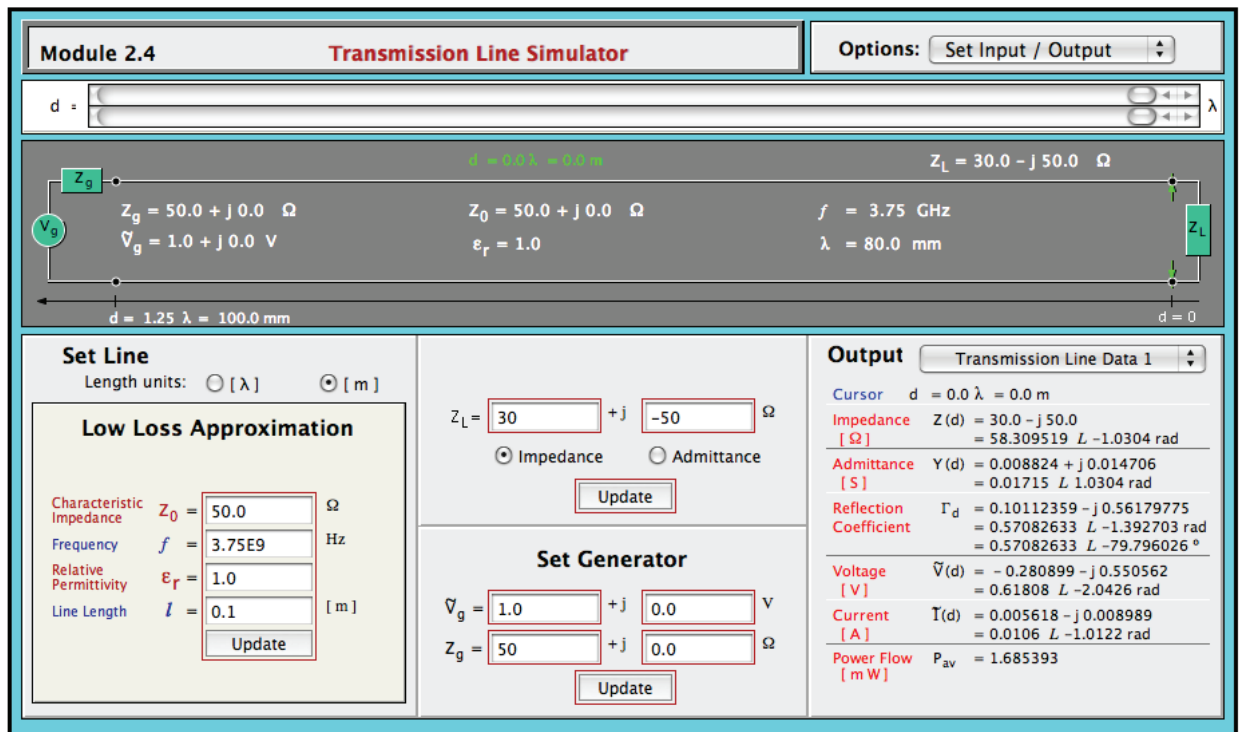
$$d_{\min} = d_{\max} - \lambda/4 = 3.11\text{ cm} - 8\text{ cm}/4 = 1.11\text{ cm}.$$

- (e) The problem statement does not specify the frequency, so in Module 2.4 we need to select the combination of  $f$  and  $\epsilon_r$  such that  $\lambda = 5\text{ cm}$ . With  $\epsilon_r$  chosen as 1,

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{8 \times 10^{-2}} = 3.75\text{ GHz}.$$

The generator parameters are irrelevant to the problem.

The results listed in the output screens are very close to those given in parts (a) through (d).



**Figure P2.19(a)**

Module 2.4
Transmission Line Simulator
Options: Set Input / Output

d =
λ

$Z_g = 50.0 + j\,0.0\ \Omega$   
 $V_g = 1.0 + j\,0.0\ \text{V}$   
 $Z_0 = 50.0 + j\,0.0\ \Omega$   
 $\epsilon_r = 1.0$   
 $f = 3.75\ \text{GHz}$   
 $\lambda = 80.0\ \text{mm}$   
 $Z_L = 30.0 - j\,50.0\ \Omega$   
 $d = 1.25\ \lambda = 100.0\ \text{mm}$   
 $d = 0$

Set Line
Length units: ☐ [λ] ☒ [m]

Low Loss Approximation
Characteristic Impedance  $Z_0 = 50.0\ \Omega$   
Frequency  $f = 3.75\text{E}9\ \text{Hz}$   
Relative Permittivity  $\epsilon_r = 1.0$   
Line Length  $l = 0.1\ \text{[m]}$   
Update

$Z_L = 30 + j\, -50\ \Omega$   
☒ Impedance ☐ Admittance  
Update

Set Generator
 $V_g = 1.0 + j\, 0.0\ \text{V}$   
 $Z_g = 50 + j\, 0.0\ \Omega$   
Update

Output
Transmission Line Data 2

SWR = 3.6601 (load)  
Amplitude of Incident Voltage Wave [V]  
 $V_0^+ = 0.0 - j\,0.5$   
= 0.5  $\angle -1.5708\ \text{rad}$

Location of First Voltage Maximum & Minimum  
 $d(\text{max}) = 0.38917\ \lambda = 31.1338\ \text{mm}$   
 $d(\text{min}) = 0.13917\ \lambda = 11.1338\ \text{mm}$

TIME-AVERAGE POWER  
 $P(\text{abs}) = 1.685393\ \text{[mW]}$  Absorbed by load  
 $P(Z_g) = 3.820225\ \text{[mW]}$  Absorbed by Zg

Figure P2.19(b)