2.31 A voltage generator with

$$v_g(t) = 5\cos(2\pi \times 10^9 t) \text{ V}$$

and internal impedance $Z_g = 50\ \Omega$ is connected to a $50\,\Omega$ lossless air-spaced transmission line. The line length is 5 cm and the line is terminated in a load with impedance $Z_L = (100 - j100)\ \Omega$. Determine:

(a) Γ at the load.

(b) Z_{in} at the input to the transmission line.

(c) The input voltage $\widetilde{V}_i$ and input current $\widetilde{I}_i$.

(d) The quantities in (a)–(c) using Modules 2.4 or 2.5.

Solution:

(a) From Eq. (2.59),

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(100 - j100) - 50}{(100 - j100) + 50} = 0.62\exp - j29.7^\circ.$$

(b) All formulas for Z_{in} require knowledge of $\beta = \omega/u_p$. Since the line is an air line, $u_p = c$, and from the expression for $v_g(t)$ we conclude $\omega = 2\pi \times 10^9$ rad/s. Therefore

$$\beta = \frac{2\pi \times 10^9 \text{ rad/s}}{3 \times 10^8 \text{ m/s}} = \frac{20\pi}{3} \text{ rad/m}.$$

Then, using Eq. (2.79),

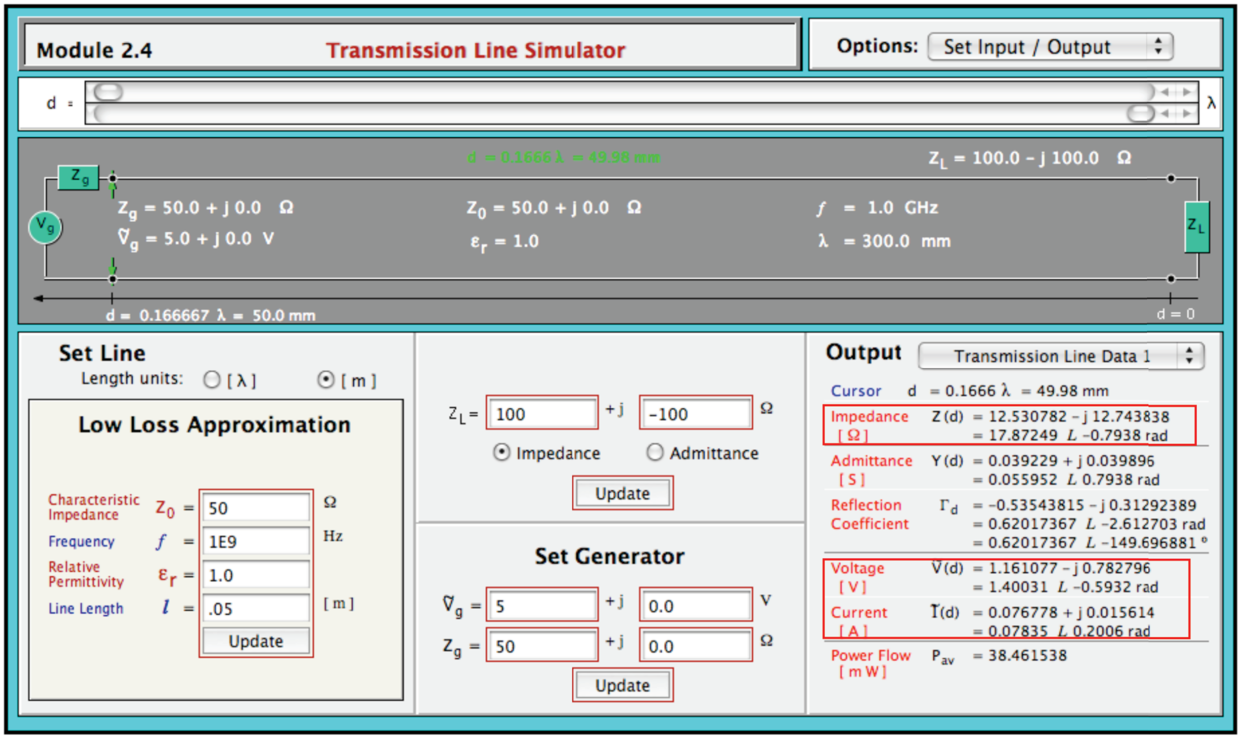
$$\begin{aligned} Z_{in} &= Z_0 \left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right) \\ &= 50 \left[\frac{(100 - j100) + j50\tan\left(\frac{20\pi}{3} \text{ rad/m} \times 5 \text{ cm}\right)}{50 + j(100 - j100)\tan\left(\frac{20\pi}{3} \text{ rad/m} \times 5 \text{ cm}\right)} \right] \\ &= 50 \left[\frac{(100 - j100) + j50\tan\left(\frac{\pi}{3} \text{ rad}\right)}{50 + j(100 - j100)\tan\left(\frac{\pi}{3} \text{ rad}\right)} \right] = (12.5 - j12.7)\ \Omega. \end{aligned}$$

(c) In phasor domain, $\widetilde{V}_g = 5 \text{ V}\exp j0^\circ$. From Eq. (2.80),

$$\widetilde{V}_i = \frac{\widetilde{V}_g Z_{in}}{Z_g + Z_{in}} = \frac{5 \times (12.5 - j12.7)}{50 + (12.5 - j12.7)} = 1.40\exp - j34.0^\circ \qquad \text{(V)},$$

and also from Eq. (2.80),

$$\widetilde{I}_i = \frac{\widetilde{V}_i}{Z_{in}} = \frac{1.4\exp - j34.0^\circ}{(12.5 - j12.7)} = 78.4\exp j11.5^\circ \qquad \text{(mA)}.$$



Module 2.4
Transmission Line Simulator
Options: Set Input / Output
d =
λ
Zg
Vg
ZL = 100.0 – j 100.0 Ω
Zg = 50.0 + j 0.0 Ω
Vg = 5.0 + j 0.0 V
Z0 = 50.0 + j 0.0 Ω
εr = 1.0
f = 1.0 GHz
λ = 300.0 mm
ZL
d = 0.166667 λ = 50.0 mm
d = 0
Set Line
Length units: [λ] [m]
Low Loss Approximation
Characteristic Impedance Z0 = 50 Ω
Frequency f = 1E9 Hz
Relative Permittivity εr = 1.0
Line Length l = .05 [m]
Update
ZL = 100 + j -100 Ω
Impedance
Admittance
Update
Set Generator
Vg = 5 + j 0.0 V
Zg = 50 + j 0.0 Ω
Update
Output
Transmission Line Data 1
Cursor d = 0.1666 λ = 49.98 mm
Impedance [Ω] Z (d) = 12.530782 – j 12.743838 = 17.87249 ∠ -0.7938 rad
Admittance [S] Y (d) = 0.039229 + j 0.039896 = 0.055952 ∠ 0.7938 rad
Reflection Coefficient Γd = -0.53543815 – j 0.31292389 = 0.62017367 ∠ -2.612703 rad = 0.62017367 ∠ -149.696881 °
Voltage [V] V(d) = 1.161077 – j 0.782796 = 1.40031 ∠ -0.5932 rad
Current [A] I(d) = 0.076778 + j 0.015614 = 0.07835 ∠ 0.2006 rad
Power Flow [m W] Pav = 38.461538