2.32 A 6 m section of 150 Ω lossless line is driven by a source with

$$\upsilon_g(t) = 5\cos(8\pi \times 10^7 t - 30°) \qquad \text{(V)}$$

and $Z_g = 150$ Ω. If the line, which has a relative permittivity $\varepsilon_r = 2.25$, is terminated in a load $Z_L = (150 - j50)$ Ω, determine:

(a) λ on the line.

(b) The reflection coefficient at the load.

(c) The input impedance.

(d) The input voltage $\widetilde{V}_i$.

(e) The time-domain input voltage $\upsilon_i(t)$.

(f) Quantities in (a) to (d) using Modules 2.4 or 2.5.

Solution:

$$\upsilon_g(t) = 5\cos(8\pi \times 10^7 t - 30°) \text{ V},$$

$$\widetilde{V}_g = 5e^{-j30°} \text{ V}.$$

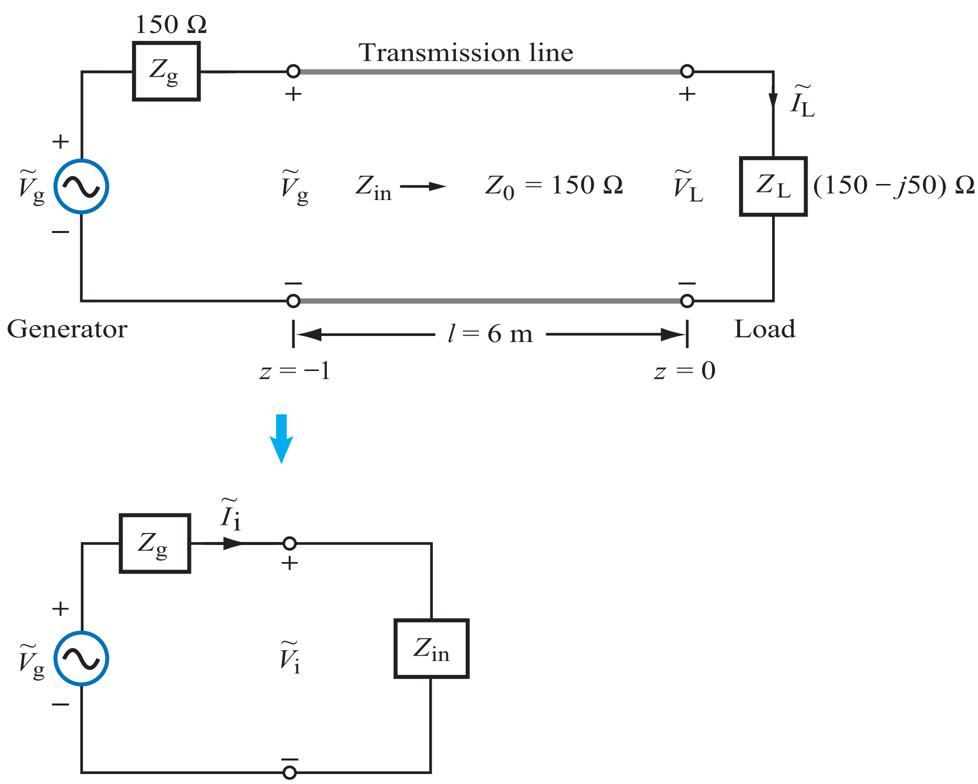


Figure P2.32 Circuit for Problem 2.32.

(a)

$$u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{2.25}} = 2 \times 10^8 \qquad \text{(m/s)},$$

$$\lambda = \frac{u_p}{f} = \frac{2\pi u_p}{\omega} = \frac{2\pi \times 2 \times 10^8}{8\pi \times 10^7} = 5 \text{ m},$$

$$\beta = \frac{\omega}{u_p} = \frac{8\pi \times 10^7}{2 \times 10^8} = 0.4\pi \qquad \text{(rad/m)},$$

$$\beta l = 0.4\pi \times 6 = 2.4\pi \qquad \text{(rad)}.$$

Since this exceeds 2π (rad), we can subtract 2π, which leaves a remainder $\beta l = 0.4\pi$ (rad).

(b) $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{150 - j50 - 150}{150 - j50 + 150} = \dfrac{-j50}{300 - j50} = 0.16\, e^{-j80.54^\circ}$.

(c)

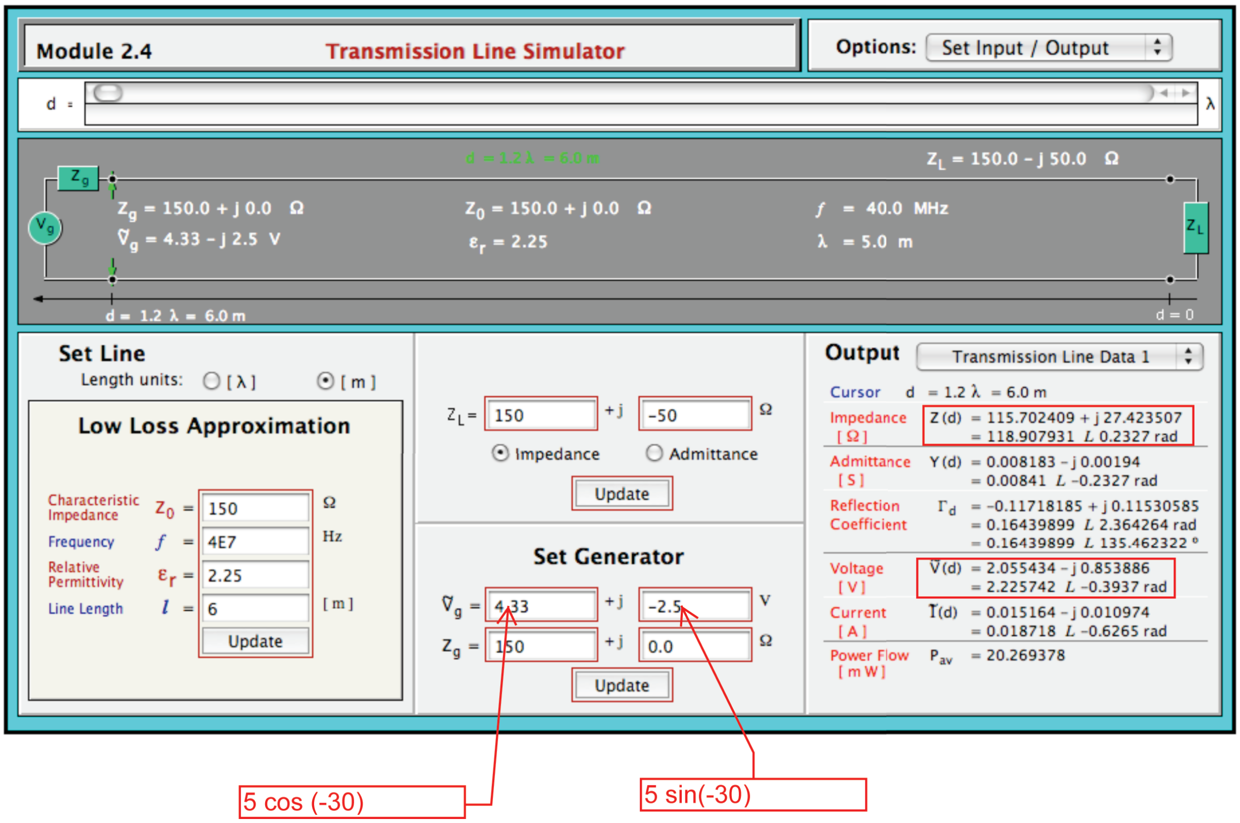
$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}\right]$$
$$= 150 \left[\frac{(150 - j50) + j150 \tan(0.4\pi)}{150 + j(150 - j50)\tan(0.4\pi)}\right] = (115.70 + j27.42)\ \Omega.$$

(d)

$$\widetilde{V}_i = \frac{\widetilde{V}_g Z_{in}}{Z_g + Z_{in}} = \frac{5e^{-j30^\circ}(115.7 + j27.42)}{150 + 115.7 + j27.42}$$
$$= 5e^{-j30^\circ}\left(\frac{115.7 + j27.42}{265.7 + j27.42}\right)$$
$$= 5e^{-j30^\circ} \times 0.44\, e^{j7.44^\circ} = 2.2\, e^{-j22.56^\circ} \qquad \text{(V)}.$$

(e)

$$\upsilon_i(t) = \mathfrak{Re}[\widetilde{V}_i e^{j\omega t}] = \mathfrak{Re}[2.2\, e^{-j22.56^\circ} e^{j\omega t}] = 2.2\cos(8\pi \times 10^7 t - 22.56^\circ) \text{ V}.$$


Module 2.4
Transmission Line Simulator
Options: Set Input / Output
d =
λ
d = 1.2 λ = 6.0 m
Z_L = 150.0 − j 50.0 Ω
Z_g = 150.0 + j 0.0 Ω
$\tilde{V}_g$ = 4.33 − j 2.5 V
Z_0 = 150.0 + j 0.0 Ω
ε_r = 2.25
f = 40.0 MHz
λ = 5.0 m
d = 1.2 λ = 6.0 m
d = 0
Set Line
Length units: [λ] [m]
Low Loss Approximation
Characteristic Impedance Z_0 = 150 Ω
Frequency f = 4E7 Hz
Relative Permittivity ε_r = 2.25
Line Length l = 6 [m]
Update
Z_L = 150 + j -50 Ω
Impedance Admittance
Update
Set Generator
$\tilde{V}_g$ = 4.33 + j -2.5 V
Z_g = 150 + j 0.0 Ω
Update
Output Transmission Line Data 1
Cursor d = 1.2 λ = 6.0 m
Impedance [Ω] Z (d) = 115.702409 + j 27.423507 = 118.907931 ∠ 0.2327 rad
Admittance [S] Y (d) = 0.008183 − j 0.00194 = 0.00841 ∠ −0.2327 rad
Reflection Coefficient Γ_d = −0.11718185 + j 0.11530585 = 0.16439899 ∠ 2.364264 rad = 0.16439899 ∠ 135.462322 °
Voltage [V] $\tilde{V}$(d) = 2.055434 − j 0.853886 = 2.225742 ∠ −0.3937 rad
Current [A] Ĩ(d) = 0.015164 − j 0.010974 = 0.018718 ∠ −0.6265 rad
Power Flow [m W] P_{av} = 20.269378
5 cos (-30)
5 sin(-30)