2.36 At an operating frequency of 300 MHz, it is desired to use a section of a lossless 50 Ω transmission line terminated in a short circuit to construct an equivalent load with reactance $X = 40$ Ω. If the phase velocity of the line is $0.75c$, what is the shortest possible line length that would exhibit the desired reactance at its input? Verify your results using Module 2.5.

Solution:

$$\beta = \omega/u_{\mathrm{p}} = \frac{(2\pi \text{ rad/cycle}) \times \left(300 \times 10^{6} \text{ cycle/s}\right)}{0.75 \times (3 \times 10^{8} \text{ m/s})} = 8.38 \text{ rad/m}.$$

On a lossless short-circuited transmission line, the input impedance is always purely imaginary; i.e., $Z_{\mathrm{in}}^{\mathrm{sc}} = jX_{\mathrm{in}}^{\mathrm{sc}}$. Solving Eq. (2.84) for the line length,

$$l = \frac{1}{\beta}\tan^{-1}\left(\frac{X_{\mathrm{in}}^{\mathrm{sc}}}{Z_0}\right) = \frac{1}{8.38 \text{ rad/m}}\tan^{-1}\left(\frac{40\ \Omega}{50\ \Omega}\right) = \frac{(0.675 + n\pi) \text{ rad}}{8.38 \text{ rad/m}},$$

for which the smallest positive solution is 8.05 cm (with $n = 0$). Since $u_{\mathrm{p}} = 0.75c$,

$$\varepsilon_{\mathrm{eff}} = \left(\frac{c}{u_{\mathrm{p}}}\right)^2 = 1.777.$$

From Module 2.5, $Z(d) = j40$ Ω when

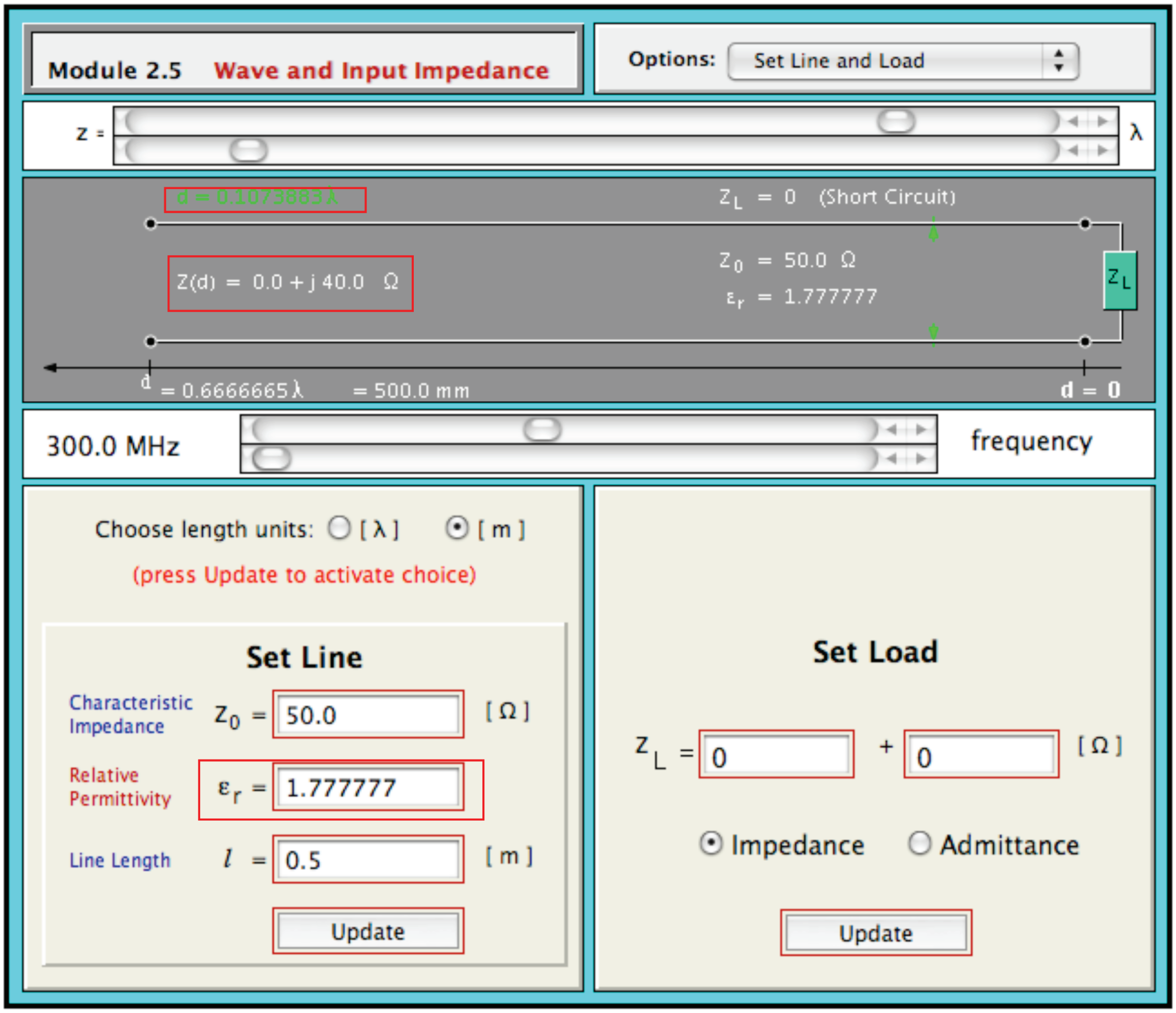
$$d = 0.107388\lambda.$$

But

$$\lambda = \frac{u_{\mathrm{p}}}{f} = \frac{0.75 \times 3 \times 10^{8}}{3 \times 10^{8}} = 0.75 \text{ m}.$$

Hence,

$$d = 0.107388 \times 0.75 = 0.0805 \text{ m} = 8.05 \text{ cm}.$$


Module 2.5 Wave and Input Impedance
Options: Set Line and Load
z =
λ
Z_L = 0 (Short Circuit)
Z(d) = 0.0 + j 40.0 Ω
Z_0 = 50.0 Ω
ε_r = 1.777777
Z_L
= 0.6666665 λ
= 500.0 mm
d = 0
300.0 MHz
frequency
Choose length units: [λ] [m]
(press Update to activate choice)
Set Line
Characteristic Impedance Z_0 = 50.0 [Ω]
Relative Permittivity ε_r = 1.777777
Line Length l = 0.5 [m]
Update
Set Load
Z_L = 0 + 0 [Ω]
Impedance
Admittance
Update