2.41 A 50 Ω lossless line of length $l = 0.375\lambda$ connects a 300 MHz generator with $\widetilde{V}_g = 300$ V and $Z_g = 50$ Ω to a load Z_L. Determine the time-domain current through the load for:

(a) $Z_L = (50 - j50)$ Ω

(b) $Z_L = 50$ Ω

(c) $Z_L = 0$ (short circuit)

For (a), verify your results by deducing the information you need from the output products generated by CD Module 2.4.

Solution:

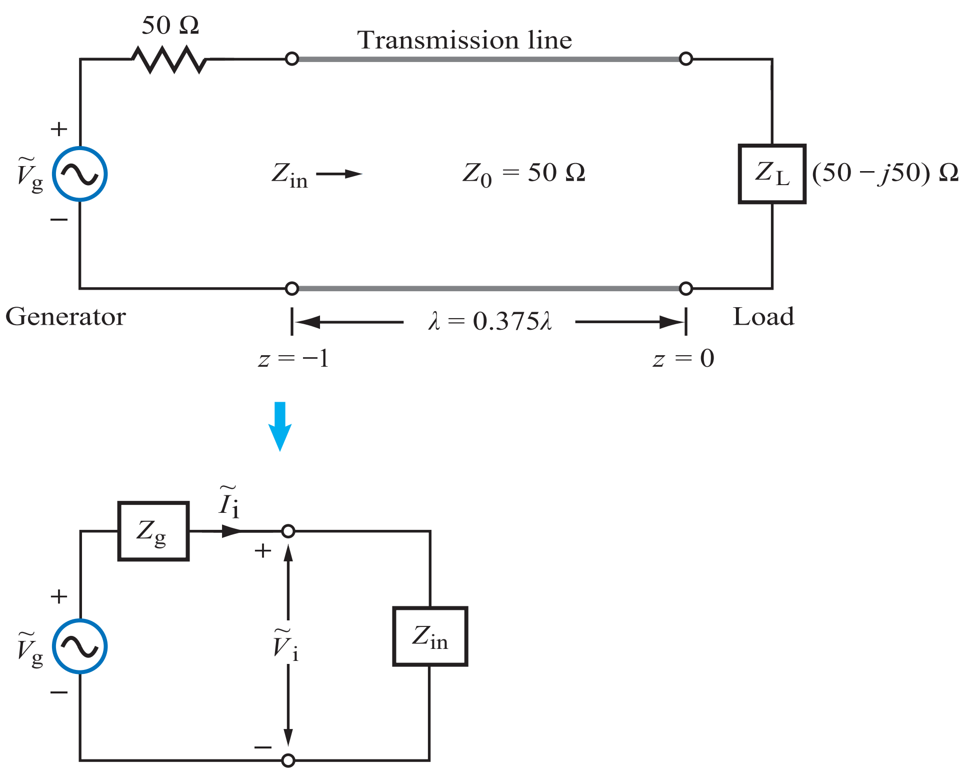

Figure P2.41 Circuit for Problem 2.41(a).

(a) $Z_L = (50 - j50)$ Ω, $\beta l = \frac{2\pi}{\lambda} \times 0.375\lambda = 2.36$ (rad) $= 135°$.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 - j50 - 50}{50 - j50 + 50} = \frac{-j50}{100 - j50} = 0.45\, e^{-j63.43°}.$$

Application of Eq. (2.79) gives:

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right] = 50 \left[\frac{(50 - j50) + j50 \tan 135°}{50 + j(50 - j50) \tan 135°}\right] = (100 + j50)\ \Omega.$$

Using Eq. (2.82) gives

$$
\begin{aligned}
V_0^+ &= \left(\frac{\widetilde{V}_\mathrm{g} Z_\mathrm{in}}{Z_\mathrm{g}+Z_\mathrm{in}}\right)\left(\frac{1}{e^{j\beta l}+\Gamma e^{-j\beta l}}\right) \\
&= \frac{300(100+j50)}{50+(100+j50)}\left(\frac{1}{e^{j135^\circ}+0.45\,e^{-j63.43^\circ}e^{-j135^\circ}}\right) \\
&= 150\,e^{-j135^\circ} \qquad \text{(V)}, \\
\tilde{I}_\mathrm{L} &= \frac{V_0^+}{Z_0}(1-\Gamma) = \frac{150\,e^{-j135^\circ}}{50}(1-0.45\,e^{-j63.43^\circ}) = 2.68\,e^{-j108.44^\circ} \qquad \text{(A)}, \\
i_\mathrm{L}(t) &= \mathfrak{Re}[\tilde{I}_\mathrm{L} e^{j\omega t}] \\
&= \mathfrak{Re}[2.68\,e^{-j108.44^\circ} e^{j6\pi\times 10^8 t}] \\
&= 2.68\cos(6\pi\times 10^8 t - 108.44^\circ) \qquad \text{(A)}.
\end{aligned}
$$

(b)

$$
\begin{aligned}
Z_\mathrm{L} &= 50\ \Omega, \\
\Gamma &= 0, \\
Z_\mathrm{in} &= Z_0 = 50\ \Omega, \\
V_0^+ &= \frac{300\times 50}{50+50}\left(\frac{1}{e^{j135^\circ}+0}\right) = 150\,e^{-j135^\circ} \qquad \text{(V)}, \\
\tilde{I}_\mathrm{L} &= \frac{V_0^+}{Z_0} = \frac{150}{50}\,e^{-j135^\circ} = 3\,e^{-j135^\circ} \qquad \text{(A)}, \\
i_\mathrm{L}(t) &= \mathfrak{Re}[3\,e^{-j135^\circ} e^{j6\pi\times 10^8 t}] = 3\cos(6\pi\times 10^8 t - 135^\circ) \qquad \text{(A)}.
\end{aligned}
$$

(c)

$$
\begin{aligned}
Z_\mathrm{L} &= 0, \\
\Gamma &= -1, \\
Z_\mathrm{in} &= Z_0\left(\frac{0+jZ_0\tan 135^\circ}{Z_0+0}\right) = jZ_0\tan 135^\circ = -j50 \qquad (\Omega), \\
V_0^+ &= \frac{300(-j50)}{50-j50}\left(\frac{1}{e^{j135^\circ}-e^{-j135^\circ}}\right) = 150\,e^{-j135^\circ} \qquad \text{(V)}, \\
\tilde{I}_\mathrm{L} &= \frac{V_0^+}{Z_0}[1-\Gamma] = \frac{150\,e^{-j135^\circ}}{50}[1+1] = 6e^{-j135^\circ} \qquad \text{(A)}, \\
i_\mathrm{L}(t) &= 6\cos(6\pi\times 10^8 t - 135^\circ) \qquad \text{(A)}.
\end{aligned}
$$

From output of Module 2.4, at $d = 0$ (load)

$$\tilde{I}(d) = 2.68\underline{/-1.89 \text{ rad}}\,,$$

which corresponds to

$$\tilde{I}(d) = 2.68\underline{/-108.29^\circ}\,.$$

The equivalent time-domain current at $f = 300$ MHz is

$$i_L(t) = 2.68\cos(6\pi \times 10^8 t - 108.29^\circ) \qquad \text{(A)}.$$