2.5 For the parallel-plate transmission line of Problem 2.4, the line parameters are given by: $R' = 1\ \Omega$/m, $L' = 167$ nH/m, $G' = 0$, and $C' = 172$ pF/m. Find α, β, u_p, and Z_0 at 1 GHz.

Solution: At 1 GHz, $\omega = 2\pi f = 2\pi \times 10^9$ rad/s. Application of (2.22) gives:

$$\begin{aligned}
\gamma &= \sqrt{(R' + j\omega L')(G' + j\omega C')} \\
&= [(1 + j2\pi \times 10^9 \times 167 \times 10^{-9})(0 + j2\pi \times 10^9 \times 172 \times 10^{-12})]^{1/2} \\
&= [(1 + j1049)(j1.1)]^{1/2} \\
&= \left[\sqrt{1 + (1049)^2}\, e^{j\tan^{-1} 1049} \times 1.1 e^{j90^\circ}\right]^{1/2}, \qquad (j = e^{j90^\circ}) \\
&= \left[1049 e^{j89.95^\circ} \times 1.1 e^{j90^\circ}\right]^{1/2} \\
&= \left[1154 e^{j179.95^\circ}\right]^{1/2} \\
&= 34 e^{j89.97^\circ} = 34\cos 89.97^\circ + j34 \sin 89.97^\circ = 0.016 + j34.
\end{aligned}$$

Hence,

$$\begin{aligned}
\alpha &= 0.016 \text{ Np/m}, \\
\beta &= 34 \text{ rad/m}.
\end{aligned}$$

$$u_p = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \frac{2\pi \times 10^9}{34} = 1.85 \times 10^8 \text{ m/s}.$$

$$\begin{aligned}
Z_0 &= \left[\frac{R' + j\omega L'}{G' + j\omega C'}\right]^{1/2} \\
&= \left[\frac{1049 e^{j89.95^\circ}}{1.1 e^{j90^\circ}}\right]^{1/2} \\
&= \left[954 e^{-j0.05^\circ}\right]^{1/2} \\
&= 31 e^{-j0.025^\circ} \approx (31 - j0.01)\ \Omega.
\end{aligned}$$