3.13 A given line is described by

$$x + 2y = 4.$$

Vector **A** starts at the origin and ends at point P on the line such that **A** is orthogonal to the line. Find an expression for **A**.

Solution: We first plot the given line. Next we find vector **B** which connects point $P_1 = (0,2)$ to $P_2 = (4,0)$, both of which are on the line:

$$\mathbf{B} = \hat{\mathbf{x}}(4-0) + \hat{\mathbf{y}}(0-2) = \hat{\mathbf{x}}4 - \hat{\mathbf{y}}2.$$

Vector **A** starts at the origin and ends on the line at P. If the x-coordinate of P is x,

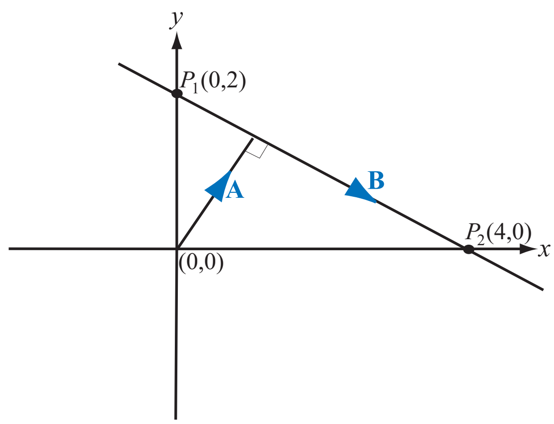

Figure P3.13 Given line and vector **A**.

then its y-coordinate has to be $(4-x)/2$ in order to be on the line. Hence P is at $(x, (4-x)/2)$. Vector **A** is

$$\mathbf{A} = \hat{\mathbf{x}}x + \hat{\mathbf{y}}\left(\frac{4-x}{2}\right).$$

But **A** is perpendicular to the line. Hence,

$$\mathbf{A} \cdot \mathbf{B} = 0,$$

$$\left[\hat{\mathbf{x}}x + \hat{\mathbf{y}}\left(\frac{4-x}{2}\right)\right] \cdot (\hat{\mathbf{x}}4 - \hat{\mathbf{y}}2) = 0,$$

$$4x - (4-x) = 0, \quad \text{or}$$

$$x = \frac{4}{5} = 0.8.$$

Hence,

$$\mathbf{A} = \hat{\mathbf{x}}0.8 + \hat{\mathbf{y}}\left(\frac{4-0.8}{2}\right) = \hat{\mathbf{x}}0.8 + \hat{\mathbf{y}}1.6.$$