3.45 The scalar function V is given by

$$V = 2\sin x + xy^2 + ye^{2z}.$$

Determine ∇V and then evaluate it at $P(0,1,2)$.

Solution: In Cartesian coordinates,

$$\begin{aligned}\nabla V &= \hat{\mathbf{x}}\,\frac{\partial V}{\partial x} + \hat{\mathbf{y}}\,\frac{\partial V}{\partial y} + \hat{\mathbf{z}}\,\frac{\partial V}{\partial z} \\ &= \hat{\mathbf{x}}(2\cos x + y^2) + \hat{\mathbf{y}}(2xy + e^{2z}) + \hat{\mathbf{z}}(2ye^{2z})\end{aligned}$$

At $P(0,1,2)$,

$$\begin{aligned}\nabla V\big|_{(0,1,2)} &= \hat{\mathbf{x}}(2\cos(0)+1) + \hat{\mathbf{y}}(0+e^4) + \hat{\mathbf{z}}\,2e^4 \\ &= \hat{\mathbf{x}}\,2 + \hat{\mathbf{y}}\,e^4 + \hat{\mathbf{z}}\,2e^4 \\ &= \hat{\mathbf{x}}\,2 + \hat{\mathbf{y}}\,54.6 + \hat{\mathbf{z}}\,109.2.\end{aligned}$$