4.20 Three infinite lines of charge, $\rho_{l_1} = 3$ (nC/m), $\rho_{l_2} = -3$ (nC/m), and $\rho_{l_3} = 3$ (nC/m), are all parallel to the z-axis. If they pass through the respective points $(0, -b)$, $(0, 0)$, and $(0, b)$ in the x–y plane, find the electric field at $(a, 0, 0)$. Evaluate your result for $a = 2$ cm and $b = 1$ cm.

Solution:

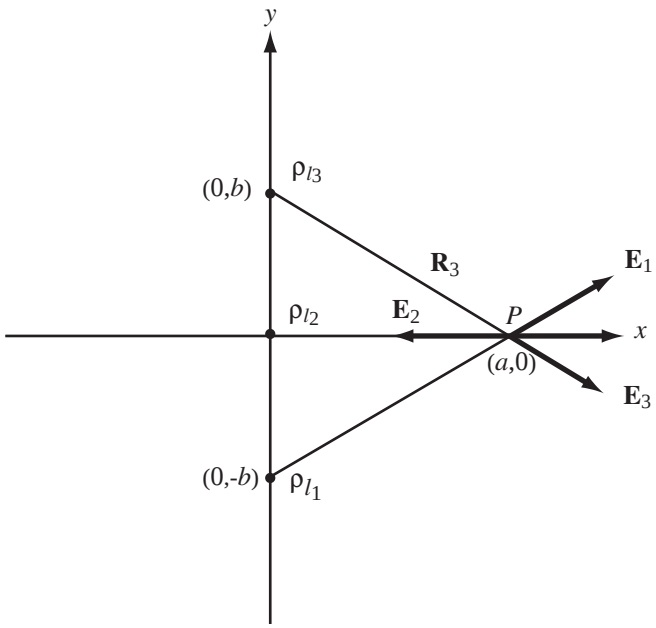

Figure P4.20 Three parallel line charges.

$$\begin{aligned}
\rho_{l_1} &= 3 \qquad \text{(nC/m)}, \\
\rho_{l_2} &= -3 \qquad \text{(nC/m)}, \\
\rho_{l_3} &= \rho_{l_1}, \\
\mathbf{E} &= \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3.
\end{aligned}$$

Components of line charges 1 and 3 along y cancel and components along x add. Hence, using Eq. (4.33),

$$\mathbf{E} = \hat{\mathbf{x}} \frac{2\rho_{l_1}}{2\pi\varepsilon_0 R_1} \cos\theta + \hat{\mathbf{x}} \frac{\rho_{l_2}}{2\pi\varepsilon_0 a}.$$

with $\cos\theta = \dfrac{a}{\sqrt{a^2+b^2}}$ and $R_1 = \sqrt{a^2+b^2}$,

$$\mathbf{E} = \frac{\hat{\mathbf{x}}\,3}{2\pi\varepsilon_0}\left[\frac{2a}{a^2+b^2} - \frac{1}{a}\right] \times 10^{-9} \qquad \text{(V/m)}.$$

For $a = 2$ cm and $b = 1$ cm,

$$\mathbf{E} = \hat{\mathbf{x}}\,1.62 \qquad \text{(kV/m)}.$$