**4.22** Given the electric flux density

$$\mathbf{D} = \hat{\mathbf{x}}2(x+y) + \hat{\mathbf{y}}(3x-2y) \qquad (\text{C/m}^2)$$

determine

**(a)** $\rho_\text{v}$ by applying Eq. (4.26).

**(b)** The total charge $Q$ enclosed in a cube 2 m on a side, located in the first octant with three of its sides coincident with the $x$-, $y$-, and $z$-axes and one of its corners at the origin.

**(c)** The total charge $Q$ in the cube, obtained by applying Eq. (4.29).

**Solution:**

**(a)** By applying Eq. (4.26)

$$\rho_\text{v} = \nabla \cdot \mathbf{D} = \frac{\partial}{\partial x}(2x+2y) + \frac{\partial}{\partial y}(3x-2y) = 0.$$

**(b)** Integrate the charge density over the volume as in Eq. (4.27):

$$Q = \int_{\mathcal{V}} \nabla \cdot \mathbf{D}\, d\mathcal{V} = \int_{x=0}^{2}\int_{y=0}^{2}\int_{z=0}^{2} 0\, dx\, dy\, dz = 0.$$

**(c)** Apply Gauss' law to calculate the total charge from Eq. (4.29)

$$Q = \oint \mathbf{D} \cdot d\mathbf{s} = F_\text{front} + F_\text{back} + F_\text{right} + F_\text{left} + F_\text{top} + F_\text{bottom},$$

$$F_\text{front} = \int_{y=0}^{2}\int_{z=0}^{2} \left(\hat{\mathbf{x}}2(x+y) + \hat{\mathbf{y}}(3x-2y)\right)\Big|_{x=2} \cdot (\hat{\mathbf{x}}\, dz\, dy)$$

$$= \int_{y=0}^{2}\int_{z=0}^{2} 2(x+y)\Big|_{x=2} dz\, dy = \left(2z\left(2y+\frac{1}{2}y^2\right)\Big|_{z=0}^{2}\right)\Big|_{y=0}^{2} = 24,$$

$$F_\text{back} = \int_{y=0}^{2}\int_{z=0}^{2} \left(\hat{\mathbf{x}}2(x+y) + \hat{\mathbf{y}}(3x-2y)\right)\Big|_{x=0} \cdot (-\hat{\mathbf{x}}\, dz\, dy)$$

$$= -\int_{y=0}^{2}\int_{z=0}^{2} 2(x+y)\Big|_{x=0} dz\, dy = -\left(zy^2\Big|_{z=0}^{2}\right)\Big|_{y=0}^{2} = -8,$$

$$F_\text{right} = \int_{x=0}^{2}\int_{z=0}^{2} \left(\hat{\mathbf{x}}2(x+y) + \hat{\mathbf{y}}(3x-2y)\right)\Big|_{y=2} \cdot (\hat{\mathbf{y}}\, dz\, dx)$$

$$= \int_{x=0}^{2}\int_{z=0}^{2}(3x-2y)\bigg|_{y=2}\,dz\,dx = \left(z\left(\frac{3}{2}x^2-4x\right)\bigg|_{z=0}^{2}\right)\bigg|_{x=0}^{2} = -4,$$

$$F_{\text{left}} = \int_{x=0}^{2}\int_{z=0}^{2}(\hat{\mathbf{x}}2(x+y)+\hat{\mathbf{y}}(3x-2y))\bigg|_{y=0}\cdot(-\hat{\mathbf{y}}\,dz\,dx)$$

$$= -\int_{x=0}^{2}\int_{z=0}^{2}(3x-2y)\bigg|_{y=0}\,dz\,dx = -\left(z\left(\frac{3}{2}x^2\right)\bigg|_{z=0}^{2}\right)\bigg|_{x=0}^{2} = -12,$$

$$F_{\text{top}} = \int_{x=0}^{2}\int_{z=0}^{2}(\hat{\mathbf{x}}2(x+y)+\hat{\mathbf{y}}(3x-2y))\bigg|_{z=2}\cdot(\hat{\mathbf{z}}\,dy\,dx)$$

$$= \int_{x=0}^{2}\int_{z=0}^{2}0\bigg|_{z=2}\,dy\,dx = 0,$$

$$F_{\text{bottom}} = \int_{x=0}^{2}\int_{z=0}^{2}(\hat{\mathbf{x}}2(x+y)+\hat{\mathbf{y}}(3x-2y))\bigg|_{z=0}\cdot(\hat{\mathbf{z}}\,dy\,dx)$$

$$= \int_{x=0}^{2}\int_{z=0}^{2}0\bigg|_{z=0}\,dy\,dx = 0.$$

Thus $Q = \oint \mathbf{D}\cdot d\mathbf{s} = 24-8-4-12+0+0 = 0.$