4.60 A coaxial capacitor consists of two concentric, conducting, cylindrical surfaces, one of radius a and another of radius b, as shown in Fig. P4.60. The insulating layer separating the two conducting surfaces is divided equally into two semi-cylindrical sections, one filled with dielectric ε_1 and the other filled with dielectric ε_2.

(a) Develop an expression for C in terms of the length l and the given quantities.

(b) Evaluate the value of C for $a = 2$ mm, $b = 6$ mm, $\varepsilon_{r_1} = 2$, $\varepsilon_{r_2} = 4$, and $l = 4$ cm.

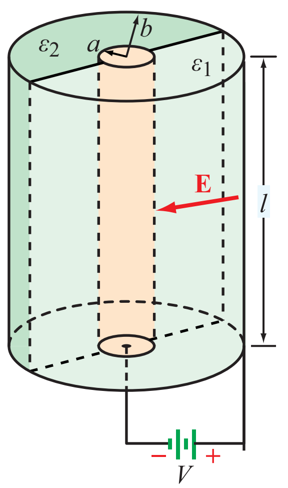

Figure P4.60 Problem 4.60.

Solution:

(a) For the indicated voltage polarity, the **E** field inside the capacitor exists in only the dielectric materials and points radially inward. Let $\mathbf{E}_1$ be the field in dielectric ε_1 and $\mathbf{E}_2$ be the field in dielectric ε_2. At the interface between the two dielectric sections, $\mathbf{E}_1$ is parallel to $\mathbf{E}_2$ and both are tangential to the interface. Since boundary conditions require that the tangential components of $\mathbf{E}_1$ and $\mathbf{E}_2$ be the same, it follows that:

$$\mathbf{E}_1 = \mathbf{E}_2 = -\hat{\mathbf{r}} E.$$

At $r = a$ (surface of inner conductor), in medium 1, the boundary condition on **D**, as stated by (4.101), leads to

$$\mathbf{D}_1 = \varepsilon_1 \mathbf{E}_1 = \hat{\mathbf{n}} \rho_{s1}$$

$$-\hat{\mathbf{r}} \varepsilon_1 E = \hat{\mathbf{r}} \rho_{s1}$$

or

$$\rho_{s1} = -\varepsilon_1 E.$$

Similarly, in medium 2

$$\rho_{s2} = -\varepsilon_2 E.$$

Thus, the **E** fields will be the same in the two dielectrics, but the charge densities will be different along the two sides of the inner conducting cylinder.

Since the same voltage applies for the two sections of the capacitor, we can treat them as two capacitors in parallel. For the capacitor half that includes dielectric ε_1, we can apply the results of Eqs. (4.114)–(4.116), but we have to keep in mind that Q is now the charge on only one half of the inner cylinder. Hence,

$$C_1 = \frac{\pi\varepsilon_1 l}{\ln(b/a)} .$$

Similarly,

$$C_2 = \frac{\pi\varepsilon_2 l}{\ln(b/a)} ,$$

and

$$C = C_1 + C_2 = \frac{\pi l(\varepsilon_1 + \varepsilon_2)}{\ln(b/a)} .$$

(b)

$$\begin{aligned} C &= \frac{\pi \times 4 \times 10^{-2}(2+4) \times 8.85 \times 10^{-12}}{\ln(6/2)} \\ &= 6.07 \text{ pF.} \end{aligned}$$