5.29 A thin current element extending between $z = -L/2$ and $z = L/2$ carries a current I along $+\hat{\mathbf{z}}$ through a circular cross-section of radius a.

(a) Find $\mathbf{A}$ at a point P located very far from the origin (assume R is so much larger than L that point P may be considered to be at approximately the same distance from every point along the current element).

(b) Determine the corresponding $\mathbf{H}$.

Solution:

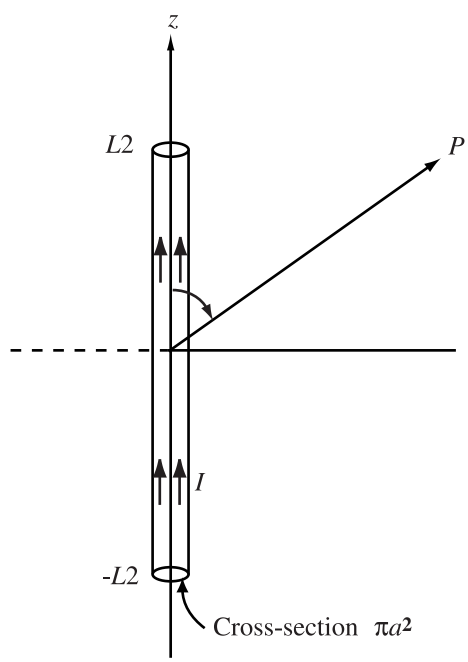

Figure P5.29 Current element of length L observed at distance $R \gg L$.

(a) Since $R \gg L$, we can assume that P is approximately equidistant from all segments of the current element. Hence, with R treated as constant, (5.65) gives

$$\mathbf{A} = \frac{\mu_0}{4\pi} \int_{\mathcal{V}'} \frac{\mathbf{J}}{R'}\, d\mathcal{V}' = \frac{\mu_0}{4\pi R} \int_{\mathcal{V}'} \hat{\mathbf{z}} \frac{I}{(\pi a^2)} \pi a^2 \, dz = \hat{\mathbf{z}} \frac{\mu_0 I}{4\pi R} \int_{-L/2}^{L/2} dz = \hat{\mathbf{z}} \frac{\mu_0 IL}{4\pi R}\,.$$

(b)

$$\mathbf{H} = \frac{1}{\mu_0} \nabla \times \mathbf{A}$$
$$= \frac{1}{\mu_0} \left[\hat{\mathbf{x}} \frac{\partial A_z}{\partial y} - \hat{\mathbf{y}} \frac{\partial A_z}{\partial x} \right]$$

$$
\begin{aligned}
&= \frac{1}{\mu_0}\left\{\hat{\mathbf{x}}\,\frac{\partial}{\partial y}\left[\frac{\mu_0 IL}{4\pi}\left(\frac{1}{\sqrt{x^2+y^2+z^2}}\right)\right] - \hat{\mathbf{y}}\,\frac{\partial}{\partial x}\left[\frac{\mu_0 IL}{4\pi}\left(\frac{1}{\sqrt{x^2+y^2+z^2}}\right)\right]\right\} \\
&= \frac{IL}{4\pi}\left[\frac{-\hat{\mathbf{x}}y+\hat{\mathbf{y}}x}{(x^2+y^2+z^2)^{3/2}}\right].
\end{aligned}
$$