5.32 The x–y plane separates two magnetic media with magnetic permeabilities μ_1 and μ_2 (Fig. P5.32). If there is no surface current at the interface and the magnetic field in medium 1 is

$$\mathbf{H}_1 = \hat{\mathbf{x}} H_{1x} + \hat{\mathbf{y}} H_{1y} + \hat{\mathbf{z}} H_{1z}$$

find:

(a) $\mathbf{H}_2$

(b) θ_1 and θ_2

(c) Evaluate $\mathbf{H}_2$, θ_1, and θ_2 for $H_{1x} = 2$ (A/m), $H_{1y} = 0$, $H_{1z} = 4$ (A/m), $\mu_1 = \mu_0$, and $\mu_2 = 4\mu_0$

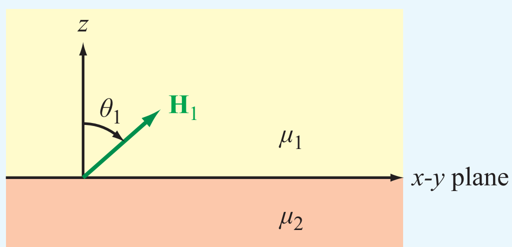

Figure P5.32 Adjacent magnetic media (Problem 5.32).

Solution:

(a) From (5.80),

$$\mu_1 H_{1\mathrm{n}} = \mu_2 H_{2\mathrm{n}},$$

and in the absence of surface currents at the interface, (5.85) states

$$H_{1\mathrm{t}} = H_{2\mathrm{t}}.$$

In this case, $H_{1z} = H_{1\mathrm{n}}$, and H_{1x} and H_{1y} are tangential fields. Hence,

$$\mu_1 H_{1z} = \mu_2 H_{2z},$$
$$H_{1x} = H_{2x},$$
$$H_{1y} = H_{2y},$$

and

$$\mathbf{H}_2 = \hat{\mathbf{x}} H_{1x} + \hat{\mathbf{y}} H_{1y} + \hat{\mathbf{z}} \frac{\mu_1}{\mu_2} H_{1z}.$$

(b)

$$H_{1\text{t}} = \sqrt{H_{1x}^2 + H_{1y}^2},$$

$$\tan\theta_1 = \frac{H_{1\text{t}}}{H_{1z}} = \frac{\sqrt{H_{1x}^2 + H_{1y}^2}}{H_{1z}},$$

$$\tan\theta_2 = \frac{H_{2\text{t}}}{H_{2z}} = \frac{\sqrt{H_{1x}^2 + H_{1y}^2}}{\frac{\mu_1}{\mu_2} H_{1z}} = \frac{\mu_2}{\mu_1}\tan\theta_1.$$

(c)

$$\mathbf{H}_2 = \hat{\mathbf{x}}2 + \hat{\mathbf{z}}\frac{1}{4}\cdot 4 = \hat{\mathbf{x}}2 + \hat{\mathbf{z}} \qquad \text{(A/m)},$$

$$\theta_1 = \tan^{-1}\left(\frac{2}{4}\right) = 26.56^\circ,$$

$$\theta_2 = \tan^{-1}\left(\frac{2}{1}\right) = 63.44^\circ.$$