5.38 A solenoid with a length of 20 cm and a radius of 5 cm consists of 400 turns and carries a current of 12 A. If $z = 0$ represents the midpoint of the solenoid, generate a plot for $|\mathbf{H}(z)|$ as a function of z along the axis of the solenoid for the range -20 cm $\leq z \leq 20$ cm in 1-cm steps.

Solution:

Figure P5.38 Problem 5.38.

Let the length of the solenoid be $l = 20$ cm. From Eq. (5.88a) and Eq. (5.88b), $z = a\tan\theta$ and $a^2 + t^2 = a^2\sec^2\theta$, which implies that $z/\sqrt{z^2+a^2} = \sin\theta$. Generalizing this to an arbitrary observation point z' on the axis of the solenoid, $(z-z')/\sqrt{(z-z')^2+a^2} = \sin\theta$. Using this in Eq. (5.89),

$$\begin{aligned}
\vec{H}(0,0,z') = \frac{\mathbf{B}}{\mu} &= \hat{\mathbf{z}}\frac{nI}{2}(\sin\theta_2 - \sin\theta_1) \\
&= \hat{\mathbf{z}}\frac{nI}{2}\left(\frac{l/2-z'}{\sqrt{(l/2-z')^2+a^2}} - \frac{-l/2-z'}{\sqrt{(-l/2-z')^2+a^2}}\right) \\
&= \hat{\mathbf{z}}\frac{nI}{2}\left(\frac{l/2-z'}{\sqrt{(l/2-z')^2+a^2}} + \frac{l/2+z'}{\sqrt{(l/2+z')^2+a^2}}\right) \qquad \text{(A/m)}.
\end{aligned}$$

A plot of the magnitude of this function of z' with $a = 5$ cm, $n = 400$ turns$/20$ cm $=$ $20,000$ turns/m, and $I = 12$ A appears in Fig. P5.38.