7.1 The magnetic field of a wave propagating through a certain nonmagnetic material is given by

$$\mathbf{H} = \hat{\mathbf{z}}\, 30\cos(10^8 t - 0.5y) \qquad \text{(mA/m)}$$

Find the following:

(a) The direction of wave propagation.

(b) The phase velocity.

(c) The wavelength in the material.

(d) The relative permittivity of the material.

(e) The electric field phasor.

Solution:

(a) Positive y-direction.

(b) $\omega = 10^8$ rad/s, $k = 0.5$ rad/m.

$$u_p = \frac{\omega}{k} = \frac{10^8}{0.5} = 2 \times 10^8 \text{ m/s}.$$

(c) $\lambda = 2\pi/k = 2\pi/0.5 = 12.6$ m.

(d) $\varepsilon_r = \left(\dfrac{c}{u_p}\right)^2 = \left(\dfrac{3 \times 10^8}{2 \times 10^8}\right)^2 = 2.25.$

(e) From Eq. (7.39b),

$$\widetilde{\mathbf{E}} = -\eta \hat{\mathbf{k}} \times \widetilde{\mathbf{H}},$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{120\pi}{\sqrt{\varepsilon_r}} = \frac{120\pi}{1.5} = 251.33 \qquad (\Omega),$$

$$\hat{\mathbf{k}} = \hat{\mathbf{y}}, \quad \text{and} \quad \widetilde{\mathbf{H}} = \hat{\mathbf{z}} 30 e^{-j0.5y} \times 10^{-3} \qquad \text{(A/m)}.$$

Hence,

$$\widetilde{\mathbf{E}} = -251.33\hat{\mathbf{y}} \times \hat{\mathbf{z}} 30 e^{-j0.5y} \times 10^{-3} = -\hat{\mathbf{x}} 7.54 e^{-j0.5y} \qquad \text{(V/m)},$$

and

$$\mathbf{E}(y,t) = \Re\mathrm{e}(\widetilde{\mathbf{E}} e^{j\omega t}) = -\hat{\mathbf{x}} 7.54\cos(10^8 t - 0.5y) \qquad \text{(V/m)}.$$