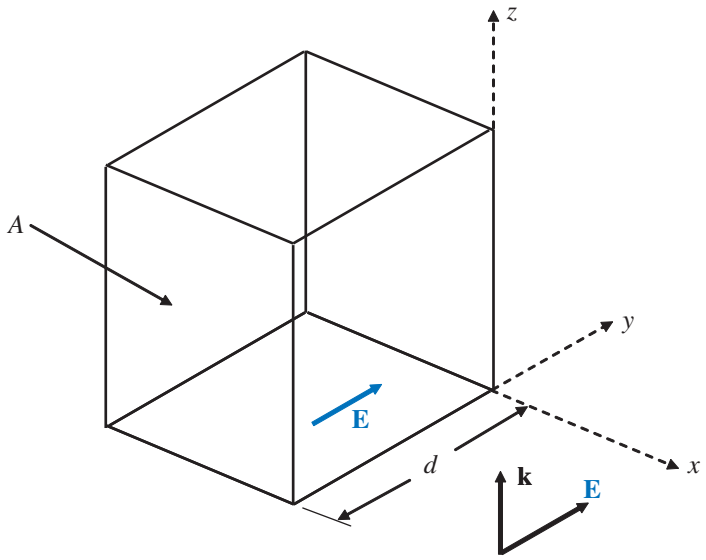
7.27 At 2 GHz, the conductivity of meat is on the order of 1 (S/m). When a material is placed inside a microwave oven and the field is activated, the presence of the electromagnetic fields in the conducting material causes energy dissipation in the material in the form of heat.

(a) Develop an expression for the time-average power per mm^3 dissipated in a material of conductivity σ if the peak electric field in the material is E_0.

(b) Evaluate the result for an electric field $E_0 = 4 \times 10^4$ (V/m).

Solution:

(a) Let us consider a small volume of the material in the shape of a box of length d and cross sectional area A. Let us assume the microwave oven creates a wave traveling along the z direction with **E** along y, as shown.



Along y, the **E** field will create a voltage difference across the length of the box V, where

$$V = Ed.$$

Conduction current through the cross sectional area A is

$$I = JA = \sigma EA.$$

Hence, the instantaneous power is

$$P = IV = \sigma E^2(Ad)$$

$$= \sigma E^2 v,$$

where $v = Ad$ is the small volume under consideration. The power per mm^3 is obtained by setting $v = (10^{-3})^3$,

$$P' = \frac{P}{10^{-9}} = \sigma E^2 \times 10^{-9} \qquad (\text{W/mm}^3).$$

As a time harmonic signal, $E = E_0 \cos \omega t$. The time average dissipated power is

$$\begin{aligned} P'_{\text{av}} &= \left[\frac{1}{T} \int_0^T \sigma E_0^2 \cos^2 \omega t \, dt \right] \times 10^{-9} \\ &= \frac{1}{2} \sigma E_0^2 \times 10^{-9} \qquad (\text{W/mm}^3). \end{aligned}$$

(b)

$$P'_{\text{av}} = \frac{1}{2} \times 1 \times (4 \times 10^4) 2 \times 10^{-9} = 0.8 \qquad (\text{W/mm}^3).$$