

8.3 A plane wave traveling in a medium with $\epsilon_{r1} = 9$ is normally incident upon a second medium with $\epsilon_{r2} = 4$. Both media are made of nonmagnetic, non-conducting materials. If the magnetic field of the incident plane wave is given by

$$\mathbf{H}^i = \hat{\mathbf{z}} 2 \cos(2\pi \times 10^9 t - ky) \quad (\text{A/m}).$$

- (a) Obtain time-domain expressions for the electric and magnetic fields in each of the two media.
- (b) Determine the average power densities of the incident, reflected, and transmitted waves.

Solution:

(a) In medium 1,

$$u_p = \frac{c}{\sqrt{\epsilon_{r1}}} = \frac{3 \times 10^8}{\sqrt{9}} = 1 \times 10^8 \quad (\text{m/s}),$$

$$k_1 = \frac{\omega}{u_p} = \frac{2\pi \times 10^9}{1 \times 10^8} = 20\pi \quad (\text{rad/m}),$$

$$\mathbf{H}^i = \hat{\mathbf{z}} 2 \cos(2\pi \times 10^9 t - 20\pi y) \quad (\text{A/m}),$$

$$\eta_1 = \frac{\eta_0}{\sqrt{\epsilon_{r1}}} = \frac{377}{3} = 125.67 \, \Omega,$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\epsilon_{r2}}} = \frac{377}{2} = 188.5 \, \Omega,$$

$$\begin{aligned} \mathbf{E}^i &= -\hat{\mathbf{x}} 2\eta_1 \cos(2\pi \times 10^9 t - 20\pi y) \\ &= -\hat{\mathbf{x}} 251.34 \cos(2\pi \times 10^9 t - 20\pi y) \quad (\text{V/m}), \end{aligned}$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{188.5 - 125.67}{188.5 + 125.67} = 0.2,$$

$$\tau = 1 + \Gamma = 1.2,$$

$$\begin{aligned} \mathbf{E}^r &= -\hat{\mathbf{x}} 251.34 \times 0.2 \cos(2\pi \times 10^9 t + 20\pi y) \\ &= -\hat{\mathbf{x}} 50.27 \cos(2\pi \times 10^9 t + 20\pi y) \quad (\text{V/m}), \end{aligned}$$

$$\begin{aligned} \mathbf{H}^r &= -\hat{\mathbf{z}} \frac{50.27}{\eta_1} \cos(2\pi \times 10^9 t + 20\pi y) \\ &= -\hat{\mathbf{z}} 0.4 \cos(2\pi \times 10^9 t + 20\pi y) \quad (\text{A/m}), \end{aligned}$$

$$\begin{aligned} \mathbf{E}_1 &= \mathbf{E}^i + \mathbf{E}^r \\ &= -\hat{\mathbf{x}} [251.34 \cos(2\pi \times 10^9 t - 20\pi y) + 50.27 \cos(2\pi \times 10^9 t + 20\pi y)] \quad (\text{V/m}), \end{aligned}$$

$$\mathbf{H}_1 = \mathbf{H}^i + \mathbf{H}^r = \hat{\mathbf{z}} [2 \cos(2\pi \times 10^9 t - 20\pi y) - 0.4 \cos(2\pi \times 10^9 t + 20\pi y)] \quad (\text{A/m}).$$

In medium 2,

$$\begin{aligned}k_2 &= \sqrt{\frac{\epsilon_2}{\epsilon_1}} k_1 = \sqrt{\frac{4}{9}} \times 20\pi = \frac{40\pi}{3} \quad (\text{rad/m}), \\ \mathbf{E}_2 = \mathbf{E}^t &= -\hat{\mathbf{x}} 251.34 \tau \cos\left(2\pi \times 10^9 t - \frac{40\pi y}{3}\right) \\ &= -\hat{\mathbf{x}} 301.61 \cos\left(2\pi \times 10^9 t - \frac{40\pi y}{3}\right) \quad (\text{V/m}), \\ \mathbf{H}_2 = \mathbf{H}^t &= \hat{\mathbf{z}} \frac{301.61}{\eta_2} \cos\left(2\pi \times 10^9 t - \frac{40\pi y}{3}\right) \\ &= \hat{\mathbf{z}} 1.6 \cos\left(2\pi \times 10^9 t - \frac{40\pi y}{3}\right) \quad (\text{A/m}).\end{aligned}$$

(b)

$$\begin{aligned}\mathbf{S}_{\text{av}}^i &= \hat{\mathbf{y}} \frac{|E_0|^2}{2\eta_1} = \hat{\mathbf{y}} \frac{(251.34)^2}{2 \times 125.67} = \hat{\mathbf{y}} 251.34 \quad (\text{W/m}^2), \\ \mathbf{S}_{\text{av}}^r &= -\hat{\mathbf{y}} |\Gamma|^2 (251.34) = \hat{\mathbf{y}} 10.05 \quad (\text{W/m}^2), \\ \mathbf{S}_{\text{av}}^t &= \hat{\mathbf{y}} (251.34 - 10.05) = \hat{\mathbf{y}} 241.29 \quad (\text{W/m}^2).\end{aligned}$$
