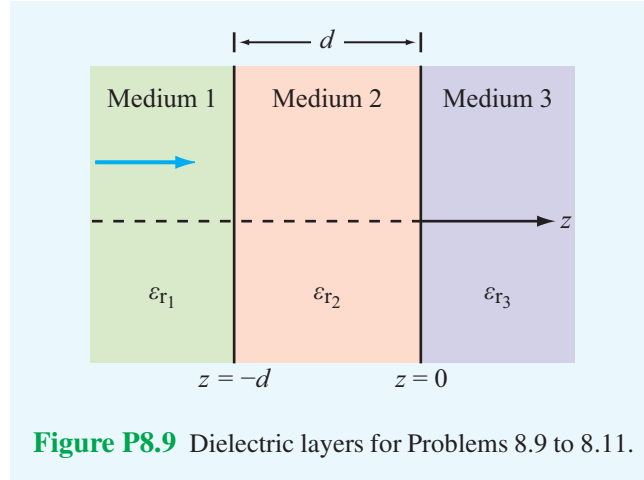


**8.9** The three regions shown in Fig. P8.9 contain perfect dielectrics. For a wave in medium 1, incident normally upon the boundary at  $z = -d$ , what combination of  $\epsilon_{r2}$  and  $d$  produces no reflection? Express your answers in terms of  $\epsilon_{r1}$ ,  $\epsilon_{r3}$  and the oscillation frequency of the wave,  $f$ .



**Figure P8.9** Dielectric layers for Problems 8.9 to 8.11.

**Solution:** By analogy with the transmission-line case, there will be no reflection at  $z = -d$  if medium 2 acts as a quarter-wave transformer, which requires that

$$d = \frac{\lambda_2}{4}$$

and

$$\eta_2 = \sqrt{\eta_1 \eta_3}.$$

The second condition may be rewritten as

$$\frac{\eta_0}{\sqrt{\epsilon_{r2}}} = \left[ \frac{\eta_0}{\sqrt{\epsilon_{r1}}} \frac{\eta_0}{\sqrt{\epsilon_{r3}}} \right]^{1/2}, \quad \text{or} \quad \epsilon_{r2} = \sqrt{\epsilon_{r1} \epsilon_{r3}},$$

$$\lambda_2 = \frac{\lambda_0}{\sqrt{\epsilon_{r2}}} = \frac{c}{f \sqrt{\epsilon_{r2}}} = \frac{c}{f (\epsilon_{r1} \epsilon_{r3})^{1/4}},$$

and

$$d = \frac{c}{4f (\epsilon_{r1} \epsilon_{r3})^{1/4}}.$$