8.15 A 5-MHz plane wave with electric field amplitude of 10 (V/m) is normally incident in air onto the plane surface of a semi-infinite conducting material with $\varepsilon_r = 4$, $\mu_r = 1$, and $\sigma = 100$ (S/m). Determine the average power dissipated (lost) per unit cross-sectional area in a 2-mm penetration of the conducting medium.

Solution: For convenience, let us choose $\mathbf{E}^i$ to be along $\hat{\mathbf{x}}$ and the incident direction to be $+\hat{\mathbf{z}}$. With

$$k_1 = \frac{\omega}{c} = \frac{2\pi \times 5 \times 10^6}{3 \times 10^8} = \frac{\pi}{30} \qquad \text{(rad/m)},$$

we have

$$\mathbf{E}^i = \hat{\mathbf{x}}\, 10 \cos\left(\pi \times 10^7 t - \frac{\pi}{30} z\right) \qquad \text{(V/m)},$$

$$\eta_1 = \eta_0 = 377\ \Omega.$$

From Table 7-1,

$$\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega \varepsilon_r \varepsilon_0} = \frac{100 \times 36\pi}{\pi \times 10^7 \times 4 \times 10^{-9}} = 9 \times 10^4,$$

which makes the material a good conductor, for which

$$\alpha_2 = \sqrt{\pi f \mu \sigma} = \sqrt{\pi \times 5 \times 10^6 \times 4\pi \times 10^{-7} \times 100} = 44.43 \qquad \text{(Np/m)},$$

$$\beta_2 = 44.43 \qquad \text{(rad/m)},$$

$$\eta_{c_2} = (1+j)\frac{\alpha_2}{\sigma} = (1+j)\frac{44.43}{100} = 0.44\,(1+j)\ \Omega.$$

According to the expression for $\mathbf{S}_{av_2}$ given in the answer to Exercise 8.3,

$$\mathbf{S}_{av_2} = \hat{\mathbf{z}}\, |\tau|^2 \frac{|E_0^i|^2}{2} e^{-2\alpha_2 z}\, \mathfrak{Re}\left(\frac{1}{\eta_{c_2}^*}\right).$$

The power lost is equal to the difference between $\mathbf{S}_{av_2}$ at $z = 0$ and $\mathbf{S}_{av_2}$ at $z = 2$ mm. Thus,

$$\begin{aligned}
P' &= \text{power lost per unit cross-sectional area} \\
&= S_{av_2}(0) - S_{av_2}(z = 2\ \text{mm}) \\
&= |\tau|^2 \frac{|E_0^i|^2}{2}\, \mathfrak{Re}\left(\frac{1}{\eta_{c_2}^*}\right) [1 - e^{-2\alpha_2 z_1}]
\end{aligned}$$

where $z_1 = 2$ mm.

$$\begin{aligned}
\tau &= 1 + \Gamma \\
&= 1 + \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 1 + \frac{0.44\,(1+j) - 377}{0.44\,(1+j) + 377} \approx 0.0023\,(1+j) = 3.3 \times 10^{-3} e^{j45^\circ}.
\end{aligned}$$

$$\mathfrak{Re}\left(\frac{1}{\eta_{c_2}^*}\right) = \mathfrak{Re}\left(\frac{1}{0.44\,(1+j)^*}\right)$$

$$= \mathfrak{Re}\left(\frac{1}{0.44\,(1-j)}\right) = \mathfrak{Re}\left(\frac{1+j}{0.44\times 2}\right) = \frac{1}{0.88} = 1.14,$$

$$P' = (3.3\times 10^{-3})^2\,\frac{10^2}{2}\times 1.14\,[1-e^{-2\times 44.43\times 2\times 10^{-3}}] = 1.01\times 10^{-4} \qquad (\mathrm{W/m^2}).$$