8.16 A 0.5-MHz antenna carried by an airplane flying over the ocean surface generates a wave that approaches the water surface in the form of a normally incident plane wave with an electric-field amplitude of 3,000 (V/m). Seawater is characterized by $\varepsilon_r = 72$, $\mu_r = 1$, and $\sigma = 4$ (S/m). The plane is trying to communicate a message to a submarine submerged at a depth d below the water surface. If the submarine's receiver requires a minimum signal amplitude of 0.01 (μV/m), what is the maximum depth d to which successful communication is still possible?

Solution: For sea water at 0.5 MHz,

$$\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon} = \frac{4 \times 36\pi}{2\pi \times 0.5 \times 10^6 \times 72 \times 10^{-9}} = 2000.$$

Hence, sea water is a good conductor, in which case we use the following expressions from Table 7-1:

$$\begin{aligned}
\alpha_2 &= \sqrt{\pi f \mu \sigma} = \sqrt{\pi \times 0.5 \times 10^6 \times 4\pi \times 10^{-7} \times 4} = 2.81 \qquad \text{(Np/m)},\\
\beta_2 &= 2.81 \qquad \text{(rad/m)},\\
\eta_{c_2} &= (1+j)\frac{\alpha_2}{\sigma} = (1+j)\frac{2.81}{4} = 0.7\,(1+j)\ \Omega,\\
\Gamma &= \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{0.7\,(1+j) - 377}{0.7\,(1+j) + 377} = (-0.9963 + j3.7 \times 10^{-3}),\\
\tau &= 1 + \Gamma = 5.24 \times 10^{-3} e^{j44.89^\circ},\\
|E^{\rm t}| &= |\tau E_0^{\rm i} e^{-\alpha_2 d}|.
\end{aligned}$$

We need to find the depth z at which $|E^{\rm t}| = 0.01\ \mu$V/m $= 10^{-8}$ V/m.

$$\begin{aligned}
10^{-8} &= 5.24 \times 10^{-3} \times 3 \times 10^3 e^{-2.81\,d},\\
e^{-2.81\,d} &= 6.36 \times 10^{-10},\\
-2.81\,d &= \ln(6.36 \times 10^{-10}) = -21.18,
\end{aligned}$$

or

$$d = 7.54 \qquad \text{(m)}.$$