8.38 Show that for nonmagnetic media, the reflection coefficient $\Gamma_\parallel$ can be written in the following form:

$$\Gamma_\parallel = \frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)}.$$

Solution: From Eq. (8.66a), $\Gamma_\parallel$ is given by

$$\Gamma_\parallel = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = \frac{(\eta_2/\eta_1)\cos\theta_t - \cos\theta_i}{(\eta_2/\eta_1)\cos\theta_t + \cos\theta_i}.$$

For nonmagnetic media, $\mu_1 = \mu_2 = \mu_0$ and

$$\frac{\eta_2}{\eta_1} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} = \frac{n_1}{n_2}.$$

Snell's law of refraction is

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{n_1}{n_2}.$$

Hence,

$$\Gamma_\parallel = \frac{\dfrac{\sin\theta_t}{\sin\theta_i}\cos\theta_t - \cos\theta_i}{\dfrac{\sin\theta_t}{\sin\theta_i}\cos\theta_t + \cos\theta_i} = \frac{\sin\theta_t\cos\theta_t - \sin\theta_i\cos\theta_i}{\sin\theta_t\cos\theta_t + \sin\theta_i\cos\theta_i}.$$

To show that the expression for $\Gamma_\parallel$ is the same as

$$\Gamma_\parallel = \frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)},$$

we shall proceed with the latter and show that it is equal to the former.

$$\frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)} = \frac{\sin(\theta_t - \theta_i)\cos(\theta_t + \theta_i)}{\cos(\theta_t - \theta_i)\sin(\theta_t + \theta_i)}.$$

Using the identities (from Appendix C):

$$2\sin x\cos y = \sin(x+y) + \sin(x-y),$$

and if we let $x = \theta_t - \theta_i$ and $y = \theta_t + \theta_i$ in the numerator, while letting $x = \theta_t + \theta_i$ and $y = \theta_t - \theta_i$ in the denominator, then

$$\frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)} = \frac{\sin(2\theta_t) + \sin(-2\theta_i)}{\sin(2\theta_t) + \sin(2\theta_i)}.$$

But $\sin 2\theta = 2\sin\theta\cos\theta$, and $\sin(-\theta) = -\sin\theta$, hence,

$$\frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)} = \frac{\sin\theta_t\cos\theta_t - \sin\theta_i\cos\theta_i}{\sin\theta_t\cos\theta_t + \sin\theta_i\cos\theta_i},$$

which is the intended result.