8.41 A 50 MHz right-hand circularly polarized plane wave with an electric field modulus of 30 V/m is normally incident in air upon a dielectric medium with $\varepsilon_r = 9$ and occupying the region defined by $z \geq 0$.

(a) Write an expression for the electric field phasor of the incident wave, given that the field is a positive maximum at $z = 0$ and $t = 0$.

(b) Calculate the reflection and transmission coefficients.

(c) Write expressions for the electric field phasors of the reflected wave, the transmitted wave, and the total field in the region $z \leq 0$.

(d) Determine the percentages of the incident average power reflected by the boundary and transmitted into the second medium.

Solution:

(a)

$$k_1 = \frac{\omega}{c} = \frac{2\pi \times 50 \times 10^6}{3 \times 10^8} = \frac{\pi}{3} \text{ rad/m},$$
$$k_2 = \frac{\omega}{c}\sqrt{\varepsilon_{r_2}} = \frac{\pi}{3}\sqrt{9} = \pi \text{ rad/m}.$$

From (7.57), RHC wave traveling in $+z$ direction:

$$\widetilde{\mathbf{E}}^i = a_0(\hat{\mathbf{x}} + \hat{\mathbf{y}}\, e^{-j\pi/2})e^{-jk_1z} = a_0(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{-jk_1z}$$
$$\mathbf{E}^i(z,t) = \mathfrak{Re}\left[\widetilde{\mathbf{E}}^i e^{j\omega t}\right]$$
$$= \mathfrak{Re}\left[a_0(\hat{\mathbf{x}}\, e^{j(\omega t - k_1z)} + \hat{\mathbf{y}}\, e^{j(\omega t - k_1z - \pi/2)})\right]$$
$$= \hat{\mathbf{x}} a_0 \cos(\omega t - k_1z) + \hat{\mathbf{y}} a_0 \cos(\omega t - k_1z - \pi/2)$$
$$= \hat{\mathbf{x}} a_0 \cos(\omega t - k_1z) + \hat{\mathbf{y}} a_0 \sin(\omega t - k_1z)$$
$$|\mathbf{E}^i| = \left[a_0^2 \cos^2(\omega t - k_1z) + a_0^2 \sin^2(\omega t - k_1z)\right]^{1/2} = a_0 = 30 \text{ V/m}.$$

Hence,

$$\widetilde{\mathbf{E}}^i = 30(x_0 - jy_0)e^{-j\pi z/3} \qquad \text{(V/m)}.$$

(b)

$$\eta_1 = \eta_0 = 120\pi \quad (\Omega), \qquad \eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{r_2}}} = \frac{120\pi}{\sqrt{9}} = 40\pi \quad (\Omega).$$
$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{40\pi - 120\pi}{40\pi + 120\pi} = -0.5$$
$$\tau = 1 + \Gamma = 1 - 0.5 = 0.5.$$

(c)

$$\widetilde{\mathbf{E}}^r = \Gamma a_0(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{jk_1z}$$
$$= -0.5 \times 30(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{jk_1z}$$
$$= -15(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{j\pi z/3} \qquad \text{(V/m)}.$$

$$\begin{aligned}
\widetilde{\mathbf{E}}^{\mathrm{t}} &= \tau a_0(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{-jk_2 z} \\
&= 15(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{-j\pi z} \qquad \text{(V/m)}. \\
\widetilde{\mathbf{E}}_1 &= \widetilde{\mathbf{E}}^{\mathrm{i}} + \widetilde{\mathbf{E}}^{\mathrm{r}} \\
&= 30(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{-j\pi z/3} - 15(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{j\pi z/3} \\
&= 15(\hat{\mathbf{x}} - j\hat{\mathbf{y}})[2e^{-j\pi z/3} - e^{j\pi z/3}] \qquad \text{(V/m)}.
\end{aligned}$$

(d)

$$\%\text{ of reflected power} = 100 \times |\Gamma|^2 = 100 \times (0.5)^2 = 25\%$$

$$\%\text{ of transmitted power} = 100|\tau|^2 \frac{\eta_1}{\eta_2} = 100 \times (0.5)^2 \times \frac{120\pi}{40\pi} = 75\%.$$