

8.46 Derive Eq. (8.89b).

Solution:

We start with Eqs. (8.88a and e),

$$\begin{aligned}\frac{\partial \tilde{e}_z}{\partial y} + j\beta \tilde{e}_y &= -j\omega\mu\tilde{h}_x, \\ -j\beta\tilde{h}_x - \frac{\partial \tilde{h}_z}{\partial x} &= j\omega\epsilon\tilde{e}_y.\end{aligned}$$

To eliminate \tilde{h}_x , we multiply the top equation by β and the bottom equation by $\omega\mu$, and then we add them together. The result is:

$$\beta \frac{\partial \tilde{e}_z}{\partial y} + j\beta^2 \tilde{e}_y - \omega\mu \frac{\partial \tilde{h}_z}{\partial x} = j\omega^2\mu\epsilon\tilde{e}_y.$$

Multiplying all terms by $e^{-j\beta z}$ to convert \tilde{e}_y to \tilde{E}_y (and similarly for the other field components), and then solving for \tilde{E}_y leads to

$$\begin{aligned}\tilde{E}_y &= \frac{1}{j(\beta^2 - \omega^2\mu\epsilon)} \left(-\beta \frac{\partial \tilde{E}_z}{\partial y} + \omega\mu \frac{\partial \tilde{H}_z}{\partial x} \right) \\ &= \frac{j}{k_c^2} \left(-\beta \frac{\partial \tilde{E}_z}{\partial y} + \omega\mu \frac{\partial \tilde{H}_z}{\partial x} \right),\end{aligned}$$

where we used the relation

$$k_c^2 = \omega^2\mu\epsilon - \beta^2.$$
