

8.51 A waveguide, with dimensions $a = 1$ cm and $b = 0.7$ cm, is to be used at 20 GHz. Determine the wave impedance for the dominant mode when

- (a) the guide is empty, and
- (b) the guide is filled with polyethylene (whose $\epsilon_r = 2.25$).

Solution: For the TE₁₀ mode,

$$f_{10} = \frac{u_{p0}}{2a} = \frac{c}{2a\sqrt{\epsilon_r}} .$$

When empty,

$$f_{10} = \frac{3 \times 10^8}{2 \times 10^{-2}} = 15 \text{ GHz}.$$

When filled with polyethylene, $f_{10} = 10$ GHz.

According to Eq. (8.111),

$$Z_{\text{TE}} = \frac{\eta}{\sqrt{1 - (f_{10}/f)^2}} = \frac{\eta_0}{\sqrt{\epsilon_r} \sqrt{1 - (f_{10}/f)^2}} .$$

When empty,

$$Z_{\text{TE}} = \frac{377}{\sqrt{1 - (15/20)^2}} = 570 \, \Omega.$$

When filled,

$$Z_{\text{TE}} = \frac{377}{\sqrt{2.25} \sqrt{1 - (10/20)^2}} = 290 \, \Omega.$$
