8.52 A narrow rectangular pulse superimposed on a carrier with a frequency of 9.5 GHz was used to excite all possible modes in a hollow guide with $a = 3$ cm and $b = 2.0$ cm. If the guide is 100 m in length, how long will it take each of the excited modes to arrive at the receiving end?

Solution: With $a = 3$ cm, $b = 2$ cm, and $u_{p_0} = c = 3 \times 10^8$ m/s, application of Eq. (8.106) leads to:

$$f_{10} = 5 \text{ GHz}$$
$$f_{01} = 7.5 \text{ GHz}$$
$$f_{11} = 9.01 \text{ GHz}$$
$$f_{20} = 10 \text{ GHz}$$

Hence, the pulse with a 9.5-GHz carrier can excite the top three modes. Their group velocities can be calculated with the help of Eq. (8.114),

$$u_g = c\sqrt{1 - (f_{mn}/f)^2},$$

which gives:

$$u_g = \begin{cases} 0.85c = 2.55 \times 10^8 \text{ m/s}, & \text{for TE}_{10} \\ 0.61c = 1.84 \times 10^8 \text{ m/s}, & \text{for TE}_{01} \\ 0.32c = 0.95 \times 10^8 \text{ m/s}, & \text{for TE}_{11} \text{ and TM}_{11} \end{cases}$$

Travel time associated with these modes is:

$$T = \frac{d}{u_g} = \frac{100}{u_g} = \begin{cases} 0.39\ \mu\text{s}, & \text{for TE}_{10} \\ 0.54\ \mu\text{s}, & \text{for TE}_{01} \\ 1.05\ \mu\text{s}, & \text{for TE}_{11} \text{ and TM}_{11}. \end{cases}$$