8.11 Repeat Problem 8.10, but interchange ε_{r_1} and ε_{r_3}.

Solution: In medium 2,

$$\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_{r_2}}} = \frac{c}{f\sqrt{\varepsilon_{r_2}}} = \frac{3\times 10^8}{5\times 10^7 \times 3} = 2 \text{ m}.$$

Hence,

$$\beta_2 = \frac{2\pi}{\lambda_2} = \pi \text{ rad/m}, \qquad \beta_2 d = 1.2\pi \text{ rad}.$$

At $z = -d$, the input impedance of a transmission line with impedance Z_L is given as Eq. (2.63),

$$Z_{in}(-d) = Z_0\left(\frac{Z_L + jZ_0\tan\beta d}{Z_0 + jZ_L\tan\beta 2d}\right).$$

In the present case, $Z_0 = \eta_2 = \eta_0/\sqrt{\varepsilon_{r_2}} = \eta_0/3$, $Z_L = \eta_3 = \eta_0/\sqrt{\varepsilon_{r_1}} = \eta_0$, where $\eta_0 = 120\pi$ (Ω). Hence,

$$\begin{aligned} Z_{in}(-d) &= \eta_2\left(\frac{\eta_3 + j\eta_2\tan 1.2\pi}{\eta_2 + j\eta_3\tan 1.2\pi}\right) \\ &= \frac{\eta_0}{3}\left(\frac{1+(j/3)\tan 1.2\pi}{(1/3)+j\tan 1.2\pi}\right) \\ &= \eta_0\left(\frac{1+(j/3)\tan 1.2\pi}{1+j3\tan 1.2\pi}\right) = (0.266 - j0.337)\eta_0 = 0.43\eta_0\angle\underline{-51.7^\circ}. \end{aligned}$$

At $z = -d$,

$$\Gamma = \frac{Z_{in} - Z_1}{Z_{in} + Z_1} = \frac{0.43\angle\underline{-51.7^\circ} - \frac{1}{2}}{0.43\angle\underline{-51.7^\circ} + \frac{1}{2}} = 0.49\angle\underline{-101.1^\circ}.$$

Fraction of incident power reflected by structure is $|\Gamma|^2 = 0.24$.