8.27 A plane wave in air with

$$\widetilde{\mathbf{E}}^{\mathrm{i}} = \hat{\mathbf{y}}\, 20 e^{-j(3x+4z)} \qquad \text{(V/m)}$$

is incident upon the planar surface of a dielectric material, with $\varepsilon_{\mathrm{r}} = 4$, occupying the half-space $z \geq 0$. Determine:

(a) The polarization of the incident wave.

(b) The angle of incidence.

(c) The time-domain expressions for the reflected electric and magnetic fields.

(d) The time-domain expressions for the transmitted electric and magnetic fields.

(e) The average power density carried by the wave in the dielectric medium.

Solution:

(a) $\widetilde{\mathbf{E}}^{\mathrm{i}} = \hat{\mathbf{y}}\, 20 e^{-j(3x+4z)}$ V/m.

Since $\mathbf{E}^{\mathrm{i}}$ is along $\hat{\mathbf{y}}$, which is perpendicular to the plane of incidence, the wave is perpendicularly polarized.

(b) From Eq. (8.48a), the argument of the exponential is

$$-jk_1(x \sin\theta_{\mathrm{i}} + z\cos\theta_{\mathrm{i}}) = -j(3x+4z).$$

Hence,

$$k_1 \sin\theta_{\mathrm{i}} = 3, \qquad k_1 \cos\theta_{\mathrm{i}} = 4,$$

from which we determine that

$$\tan\theta_{\mathrm{i}} = \frac{3}{4} \qquad \text{or} \qquad \theta_{\mathrm{i}} = 36.87^\circ,$$

and

$$k_1 = \sqrt{3^2+4^2} = 5 \qquad \text{(rad/m)}.$$

Also,

$$\omega = u_{\mathrm{p}}k = ck = 3\times 10^8 \times 5 = 1.5\times 10^9 \qquad \text{(rad/s)}.$$

(c)

$$\eta_1 = \eta_0 = 377\ \Omega,$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{\mathrm{r}_2}}} = \frac{\eta_0}{2} = 188.5\ \Omega,$$

$$\theta_{\mathrm{t}} = \sin^{-1}\left[\frac{\sin\theta_{\mathrm{i}}}{\sqrt{\varepsilon_{\mathrm{r}_2}}}\right] = \sin^{-1}\left[\frac{\sin 36.87^\circ}{\sqrt{4}}\right] = 17.46^\circ,$$

$$\Gamma_\perp = \frac{\eta_2\cos\theta_{\mathrm{i}} - \eta_1\cos\theta_{\mathrm{t}}}{\eta_2\cos\theta_{\mathrm{i}} + \eta_1\cos\theta_{\mathrm{t}}} = -0.41,$$

$$\tau_\perp = 1 + \Gamma_\perp = 0.59.$$

In accordance with Eq. (8.49a), and using the relation $E_0^{\rm r} = \Gamma_\perp E_0^{\rm i}$,

$$\widetilde{\mathbf{E}}^{\rm r} = -\hat{\mathbf{y}}\, 8.2\, e^{-j(3x-4z)},$$

$$\widetilde{\mathbf{H}}^{\rm r} = -(\hat{\mathbf{x}} \cos\theta_{\rm i} + \hat{\mathbf{z}} \sin\theta_{\rm i})\, \frac{8.2}{\eta_0}\, e^{-j(3x-4z)},$$

where we used the fact that $\theta_{\rm i} = \theta_{\rm r}$ and the z-direction has been reversed.

$$\mathbf{E}^{\rm r} = \mathfrak{Re}[\widetilde{\mathbf{E}}^{\rm r} e^{j\omega t}] = -\hat{\mathbf{y}}\, 8.2 \cos(1.5\times 10^9 t - 3x + 4z) \qquad \text{(V/m)},$$

$$\mathbf{H}^{\rm r} = -(\hat{\mathbf{x}}\, 17.4 + \hat{\mathbf{z}}\, 13.06) \cos(1.5\times 10^9 t - 3x + 4z) \qquad \text{(mA/m)}.$$

(d) In medium 2,

$$k_2 = k_1 \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = 5\sqrt{4} = 20 \qquad \text{(rad/m)},$$

and

$$\theta_{\rm t} = \sin^{-1}\left[\sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \sin\theta_{\rm i}\right] = \sin^{-1}\left[\frac{1}{2}\sin 36.87^\circ\right] = 17.46^\circ$$

and the exponent of $\mathbf{E}^{\rm t}$ and $\mathbf{H}^{\rm t}$ is

$$-jk_2(x\sin\theta_{\rm t} + z\cos\theta_{\rm t}) = -j10(x\sin 17.46^\circ + z\cos 17.46^\circ) = -j(3x + 9.54z).$$

Hence,

$$\widetilde{\mathbf{E}}^{\rm t} = \hat{\mathbf{y}}\, 20 \times 0.59\, e^{-j(3x+9.54z)},$$

$$\widetilde{\mathbf{H}}^{\rm t} = (-\hat{\mathbf{x}} \cos\theta_{\rm t} + \hat{\mathbf{z}} \sin\theta_{\rm t})\, \frac{20\times 0.59}{\eta_2}\, e^{-j(3x+9.54z)}.$$

$$\mathbf{E}^{\rm t} = \mathfrak{Re}[\widetilde{\mathbf{E}}^{\rm t} e^{j\omega t}] = \hat{\mathbf{y}}\, 11.8 \cos(1.5\times 10^9 t - 3x - 9.54z) \qquad \text{(V/m)},$$

$$\begin{aligned}\mathbf{H}^{\rm t} &= (-\hat{\mathbf{x}} \cos 17.46^\circ + \hat{\mathbf{z}} \sin 17.46^\circ)\, \frac{11.8}{188.5} \cos(1.5\times 10^9 t - 3x - 9.54z) \\ &= (-\hat{\mathbf{x}}\, 59.72 + \hat{\mathbf{z}}\, 18.78) \cos(1.5\times 10^9 t - 3x - 9.54z) \qquad \text{(mA/m)}.\end{aligned}$$

(e)

$$S_{\rm av}^{\rm t} = \frac{|E_0^{\rm t}|^2}{2\eta_2} = \frac{(11.8)^2}{2\times 188.5} = 0.36 \qquad \text{(W/m}^2\text{)}.$$