

9.4 Determine the following:

- (a) The direction of maximum radiation.
- (b) Directivity.
- (c) Beam solid angle.
- (d) Half-power beamwidth in the x - z plane.

for an antenna whose normalized radiation intensity is given by

$$F(\theta, \phi) = \begin{cases} 1, & \text{for } 0 \leq \theta \leq 60^\circ \text{ and } 0 \leq \phi \leq 2\pi \\ 0, & \text{elsewhere.} \end{cases}$$

Suggestion: Sketch the pattern prior to calculating the desired quantities.

Solution: The direction of maximum radiation is a circular cone 120° wide centered around the $+\hat{z}$ axis. From Eq. (9.23),

$$\begin{aligned} D &= \frac{4\pi}{\iint_{4\pi} F d\Omega} \\ &= \frac{4\pi}{\int_0^{2\pi} \int_0^{60^\circ} \sin \theta d\theta d\phi} = \frac{4\pi}{2\pi (-\cos \theta) \big|_0^{60^\circ}} = \frac{2}{-\frac{1}{2} + 1} = 4 = 6 \text{ dB}, \\ \Omega_p &= \frac{4\pi \text{ sr}}{D} = \frac{4\pi \text{ sr}}{4} = \pi \quad (\text{sr}). \end{aligned}$$

The half power beamwidth is $\beta = 120^\circ$.
