

9.5 Repeat Problem 9.4 for an antenna with

$$F(\theta, \phi) = \begin{cases} \sin^2 \theta \cos^2 \phi & \text{for } 0 \leq \theta \leq \pi \\ & \text{and } -\pi/2 \leq \phi \leq \pi/2 \\ 0 & \text{elsewhere} \end{cases}$$

Solution: The direction of maximum radiation is the $+\hat{\mathbf{x}}$ axis (where $\theta = \pi/2$ and $\phi = 0$). From Eq. (9.23),

$$\begin{aligned} D &= \frac{4\pi}{\iint_{4\pi} F d\Omega} \\ &= \frac{4\pi}{\int_{-\pi/2}^{\pi/2} \int_0^\pi \sin^2 \theta \cos^2 \phi \sin \theta d\theta d\phi} \\ &= \frac{4\pi}{\int_{-\pi/2}^{\pi/2} \cos^2 \phi d\phi \int_0^\pi \sin^3 \theta d\theta} \\ &= \frac{4\pi}{\int_{-\pi/2}^{\pi/2} \frac{1}{2} (1 + \cos 2\phi) d\phi \int_{-1}^1 (1 - x^2) dx} \\ &= \frac{4\pi}{\left. \frac{1}{2} \left(\phi + \frac{1}{2} \sin 2\phi \right) \right|_{-\pi/2}^{\pi/2} \left. (x - x^3/3) \right|_{-1}^1} = \frac{4\pi}{\frac{1}{2}\pi(4/3)} = 6 = 7.8 \text{ dB}, \\ \Omega_p &= \frac{4\pi \text{ sr}}{D} = \frac{4\pi \text{ sr}}{6} = \frac{2}{3}\pi \quad (\text{sr}). \end{aligned}$$

In the x - z plane, $\phi = 0$ and the half power beamwidth is 90° , since

$$\sin^2(45^\circ) = \sin^2(135^\circ) = \frac{1}{2}.$$
