9.12 The normalized radiation intensity of a certain antenna is given by

$$F(\theta) = \exp(-20\theta^2) \qquad \text{for } 0 \le \theta \le \pi$$

where θ is in radians. Determine:

(a) The half-power beamwidth.

(b) The pattern solid angle.

(c) The antenna directivity.

Solution:

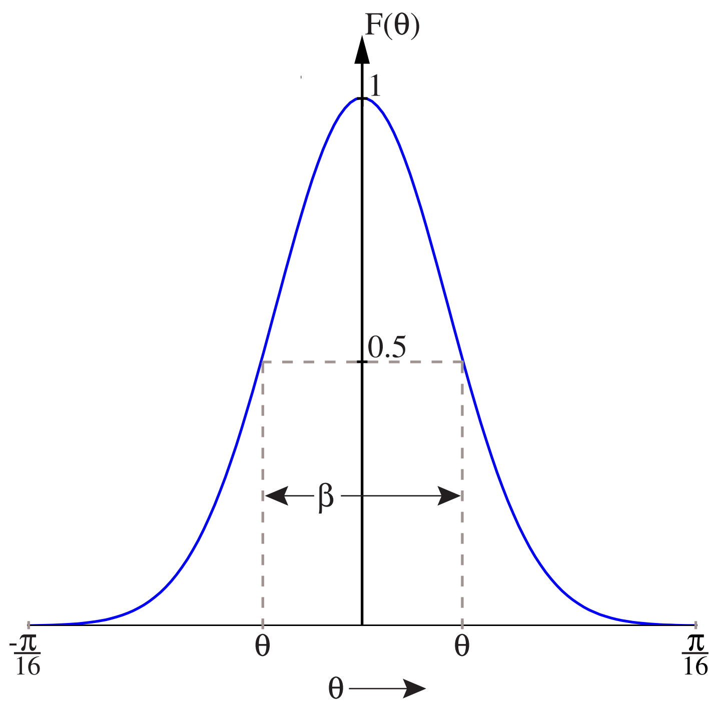

Figure P9.12 $F(\theta)$ versus θ.

(a) Since $F(\theta)$ is independent of ϕ, the beam is symmetrical about $z = 0$. Upon setting $F(\theta) = 0.5$, we have

$$F(\theta) = \exp(-20\theta^2) = 0.5,$$

$$\ln[\exp(-20\theta^2)] = \ln(0.5),$$

$$20\theta^2 = -0.693,$$

$$\theta = \pm\left(\frac{0.693}{20}\right)^{1/2} = \pm 0.186 \text{ radians}.$$

Hence, $\beta = 2 \times 0.186 = 0.372$ radians $= 21.31°$.

(b) By Eq. (9.21),

$$\begin{aligned}
\Omega_{\mathrm{p}} &= \iint_{4\pi} F(\theta) \sin\theta \, d\theta \, d\phi \\
&= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \exp(-20\theta^2) \sin\theta \, d\theta \, d\phi \\
&= 2\pi \int_0^{\pi} \exp(-20\theta^2) \sin\theta \, d\theta.
\end{aligned}$$

Numerical evaluation yields

$$\Omega_{\mathrm{p}} = 0.156 \text{ sr}.$$

(c)

$$D = \frac{4\pi}{\Omega_{\mathrm{p}}} = \frac{4\pi}{0.156} = 80.55.$$