9.16 For a short dipole with length l such that $l \ll \lambda$, instead of treating the current $\widetilde{I}(z)$ as constant along the dipole, as was done in Section 9-1, a more realistic approximation that ensures the current goes to zero at the dipole ends is to describe $\widetilde{I}(z)$ by the triangular function

$$\widetilde{I}(z) = \begin{cases} I_0(1-2z/l), & \text{for } 0 \le z \le l/2 \\ I_0(1+2z/l), & \text{for } -l/2 \le z \le 0 \end{cases}$$

as shown in Fig. P9.14. Use this current distribution to determine the following:

(a) The far-field $\widetilde{\mathbf{E}}(R,\theta,\phi)$.

(b) The power density $S(R,\theta,\phi)$.

(c) The directivity D.

(d) The radiation resistance R_{rad}.

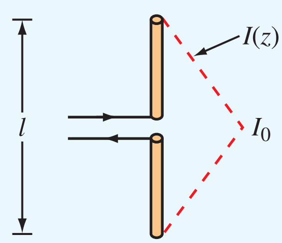

Figure P9.16 Triangular current distribution on a short dipole (Problem 9.16).

Solution:

(a) When the current along the dipole was assumed to be constant and equal to I_0, the vector potential was given by Eq. (9.3) as:

$$\widetilde{\mathbf{A}}(R) = \hat{\mathbf{z}}\,\frac{\mu_0}{4\pi}\left(\frac{e^{-jkR}}{R}\right)\int_{-l/2}^{l/2} I_0\,dz.$$

If the triangular current function is assumed instead, then I_0 in the above expression should be replaced with the given expression. Hence,

$$\widetilde{\mathbf{A}} = \hat{\mathbf{z}}\,\frac{\mu_0}{4\pi}\left(\frac{e^{-jkR}}{R}\right) I_0\left[\int_0^{l/2}\left(1-\frac{2z}{l}\right)dz + \int_{-l/2}^{0}\left(1+\frac{2z}{l}\right)dz\right] = \hat{\mathbf{z}}\,\frac{\mu_0 I_0 l}{8\pi}\left(\frac{e^{-jkR}}{R}\right),$$

which is half that obtained for the constant-current case given by Eq. (9.3). Hence, the expression given by (9.9a) need only be modified by the factor of $1/2$:

$$\widetilde{\mathbf{E}} = \hat{\boldsymbol{\theta}}\widetilde{E}_\theta = \hat{\boldsymbol{\theta}}\,\frac{jI_0 lk\eta_0}{8\pi}\left(\frac{e^{-jkR}}{R}\right)\sin\theta.$$

(b) The corresponding power density is

$$S(R,\theta) = \frac{|\widetilde{E}_\theta|^2}{2\eta_0} = \left(\frac{\eta_0 k^2 I_0^2 l^2}{128\pi^2 R^2}\right)\sin^2\theta.$$

(c) The power density is 4 times smaller than that for the constant current case, but the reduction is true for all directions. Hence, D remains unchanged at 1.5.

(d) Since $S(R,\theta)$ is 4 times smaller, the total radiated power P_{rad} is 4-times smaller. Consequently, $R_{\text{rad}} = 2P_{\text{rad}}/I_0^2$ is 4 times smaller than the expression given by Eq. (9.35); that is,

$$R_{\text{rad}} = 20\pi^2(l/\lambda)^2 \qquad (\Omega).$$