9.42 A two-element array consisting of two isotropic antennas separated by a distance d along the z axis is placed in a coordinate system whose z axis points eastward and whose x axis points toward the zenith. If a_0 and a_1 are the amplitudes of the excitations of the antennas at $z = 0$ and at $z = d$, respectively, and if δ is the phase of the excitation of the antenna at $z = d$ relative to that of the other antenna, find the array factor and plot the pattern in the x–z plane for the following:

(a) $a_0 = a_1 = 1,\ \delta = \pi/4$, and $d = \lambda/2$

(b) $a_0 = 1,\ a_1 = 2,\ \delta = 0$, and $d = \lambda$

(c) $a_0 = a_1 = 1,\ \delta = -\pi/2$, and $d = \lambda/2$

(d) $a_0 = 1,\ a_1 = 2,\ \delta = \pi/4$, and $d = \lambda/2$

(e) $a_0 = 1,\ a_1 = 2,\ \delta = \pi/2$, and $d = \lambda/4$

Solution:

(a) Employing Eq. (9.110),

$$
\begin{aligned}
F_{\mathrm{a}}(\theta) &= \left| \sum_{i=0}^{1} a_i \exp j\psi_i \exp jikd\cos\theta \right|^2 \\
&= |1+\exp j((2\pi/\lambda)(\lambda/2)\cos\theta+\pi/4)|^2 \\
&= |1+\exp j(\pi\cos\theta+\pi/4)|^2 = 4\cos^2\left[\frac{\pi}{8}(4\cos\theta+1)\right].
\end{aligned}
$$

A plot of this array factor pattern is shown in Fig. 9-42(a).

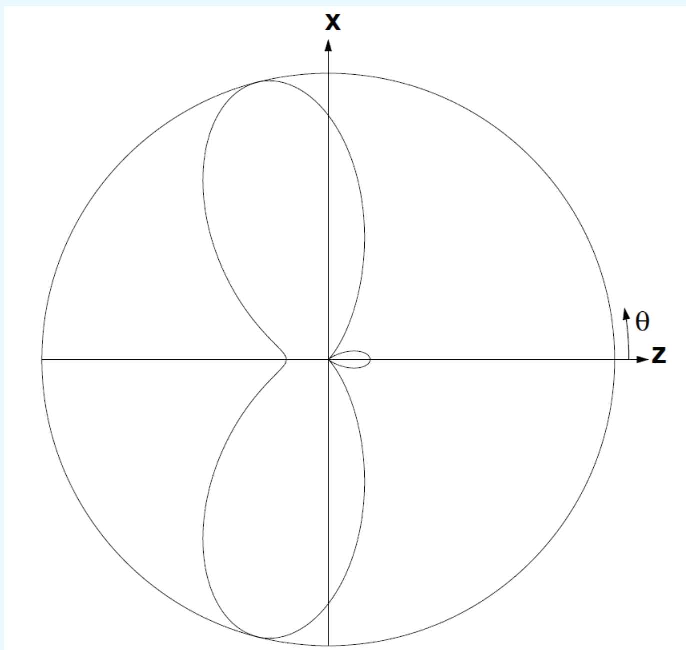

Figure P9.42 (a) Array factor in the elevation plane for Problem 9.42(a).

(b) Employing Eq. (9.110),

$$F_\mathrm{a}(\theta) = \left|\sum_{i=0}^{1} a_i \exp j\psi_i \exp jikd\cos\theta\right|^2$$

$$= |1+2\exp j((2\pi/\lambda)\lambda\cos\theta+0)|^2 = |1+2\exp j2\pi\cos\theta|^2 = 5+4\cos(2\pi\cos\theta)\,.$$

A plot of this array factor pattern is shown in Fig. 9-42(b).

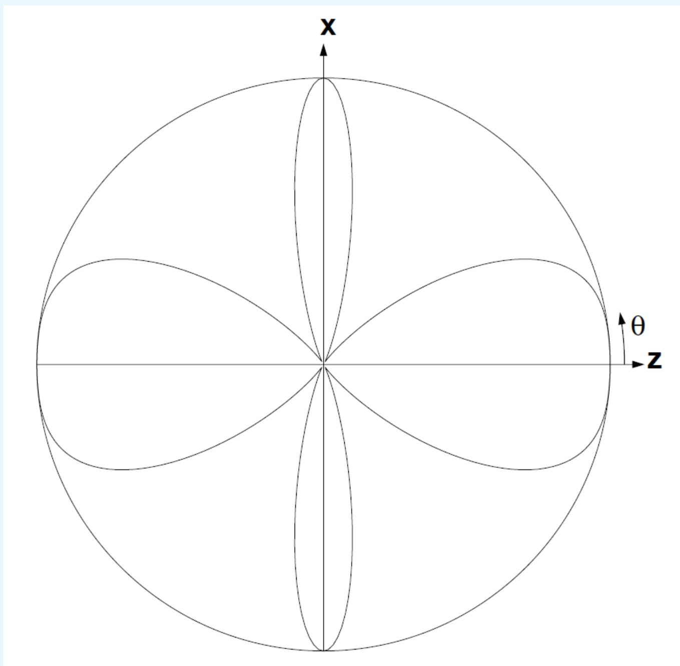

Figure P9.42 (b) Array factor in the elevation plane for Problem 9.42(b).

(c) Employing Eq. (9.110), and setting $a_0 = a_1 = 1$, $\psi = 0$, $\psi_1 = \delta = -\pi/2$ and $d = \lambda/2$, we have

$$
\begin{aligned}
F_\text{a}(\theta) &= \left| \sum_{i=0}^{1} a_i e^{j\psi_i} e^{jikd\cos\theta} \right|^2 \\
&= \left| 1 + e^{-j\pi/2} e^{j(2\pi/\lambda)(\lambda/2)\cos\theta} \right|^2 \\
&= \left| 1 + e^{j(\pi\cos\theta - \pi/2)} \right|^2 \\
&= 4\cos^2\left(\frac{\pi}{2}\cos\theta - \frac{\pi}{4}\right).
\end{aligned}
$$

A plot of the array factor is shown in Fig. 9-42(c).

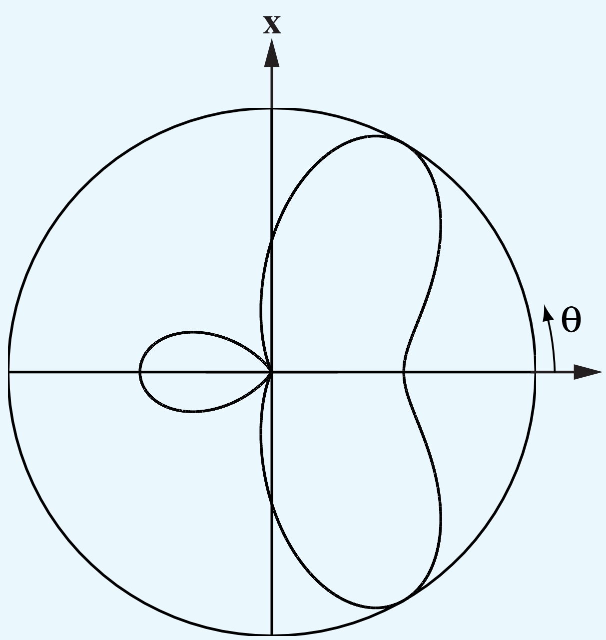

Figure P9.42 (c) Array factor in the elevation plane for Problem 9.42(c).

(d) Employing Eq. (9.110), and setting $a_0 = 1$, $a_1 = 2$, $\psi_0 = 0$, $\psi_1 = \delta = \pi/4$, and $d = \lambda/2$, we have

$$
\begin{aligned}
F_a(\theta) &= \left| \sum_{i=0}^{1} a_i e^{j\psi_i} e^{jikd\cos\theta} \right|^2 \\
&= \left| 1 + 2e^{j\pi/4} e^{j(2\pi/\lambda)(\lambda/2)\cos\theta} \right|^2 \\
&= \left| 1 + 2e^{j(\pi\cos\theta + \pi/4)} \right|^2 \\
&= 5 + 4\cos\left(\pi\cos\theta + \frac{\pi}{4}\right).
\end{aligned}
$$

A plot of the array factor is shown in Fig. 9-42(d).

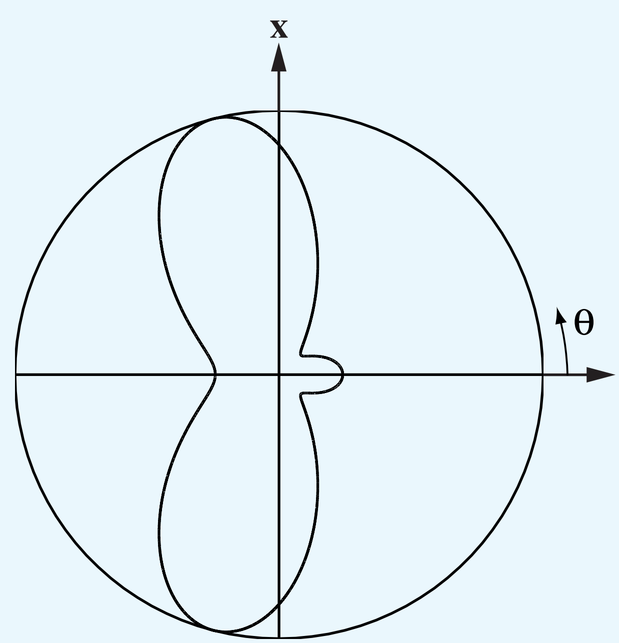

Figure P9.42 (d) Array factor in the elevation plane for Problem 9.42(d).

(e) Employing Eq. (9.110), and setting $a_0 = 1$, $a_1 = 2$, $\psi_0 = 0$, $\psi_1 = \delta = \pi/2$, and $d = \lambda/4$, we have

$$
\begin{aligned}
F_{\mathrm{a}}(\theta) &= \left| \sum_{i=0}^{1} a_i e^{j\psi_i} e^{jikd\cos\theta} \right|^2 \\
&= \left| 1 + 2e^{j\pi/2} e^{j(2\pi/\lambda)(\lambda/4)\cos\theta} \right|^2 \\
&= \left| 1 + 2e^{j(\pi\cos\theta+\pi)/2} \right|^2 \\
&= 5 + 4\cos\left(\frac{\pi}{2}\cos\theta + \frac{\pi}{2}\right) = 5 - 4\sin\left(\frac{\pi}{2}\cos\theta\right).
\end{aligned}
$$

A plot of the array factor is shown in Fig. 9-42(e).

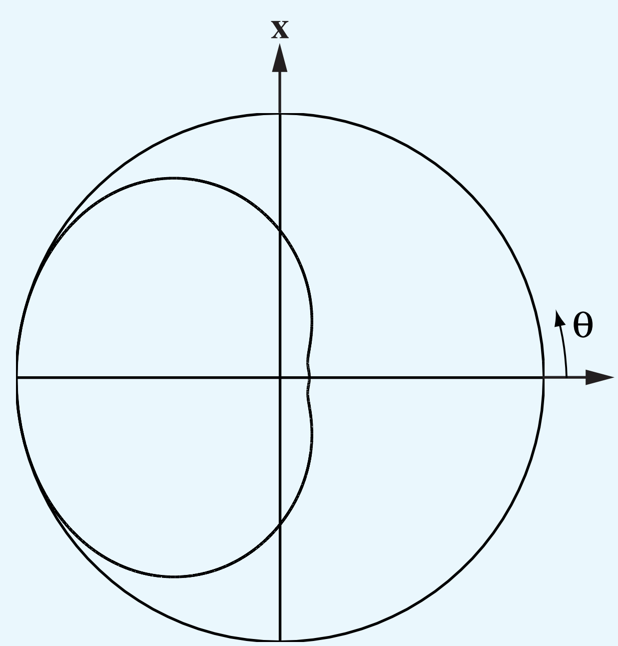

Figure P9.42 (e) Array factor in the elevation plane for Problem 9.42(e).