**9.47** Repeat Problem 9.46 but change the excitation to tapered amplitude distribution such that the amplitude of the central element is 1, the amplitudes of the next adjacent elements are both 0.5, and those of the outer elements are both 0.25.

**Solution:** From Eq. (9.101) with $N = 5$,

$$F_{\text{a}}(\theta) = \left| \sum_{i=0}^{4} A_i e^{jikd\cos\theta} \right|^2 .$$

Here,

$$kd = \frac{2\pi}{\lambda} \cdot \frac{3\lambda}{4} = \frac{3\pi}{2} .$$

Also,

$$A_i = \begin{cases} 0.25 & \text{for } i = 0, \\ 0.5 & \text{for } i = 1, \\ 1 & \text{for } i = 2, \\ 0.5 & \text{for } i = 3, \\ 0.25 & \text{for } i = 4. \end{cases}$$

Hence,

$$\begin{aligned} F_{\text{a}}(\theta) &= \left| 0.25 + 0.5e^{j(3\pi/2)\cos\theta} + e^{j(3\pi\cos\theta)} \right. \\ &\qquad \left. + 0.5e^{j(9\pi/2)\cos\theta)} + 0.25e^{j(6\pi\cos\theta)} \right|^2 \\ &= \left| e^{j3\pi\cos\theta} \left( 0.25e^{-j3\pi\cos\theta} + 0.25e^{-j(3\pi/2)\cos\theta} + 1 \right. \right. \\ &\qquad \left. \left. + 0.5e^{j(3\pi/2)\cos\theta} + 0.25e^{j3\pi\cos\theta} \right) \right|^2 \\ &= \left| \frac{1}{2}\cos(3\pi\cos\theta) + \cos\left( \frac{3\pi}{2}\cos\theta \right) + 1 \right|^2 . \end{aligned}$$

The plot of $F_{\text{a}}(\theta)$ versus $\theta$ shows that it is a maximum at $\theta = 90^\circ$, at which

$$F_{\text{a max}} = \left( \frac{1}{2} + 1 + 1 \right)^2 = \frac{25}{4} .$$

Hence,

$$F_{\text{an}} = \frac{F_{\text{a}}}{F_{\text{a max}}} = \frac{4}{25} \left[ \frac{1}{2}\cos(3\pi\cos\theta) + \cos\left( \frac{3\pi}{2}\cos\theta \right) + 1 \right]^2 .$$

From the plot, we determine that $F_{\text{an}} = 0.5$ at

$$\theta_2 = 99^\circ .$$

Hence,

$$\beta = 2(\theta_2 - 90^\circ) = 2(99^\circ - 90^\circ) = 18^\circ .$$

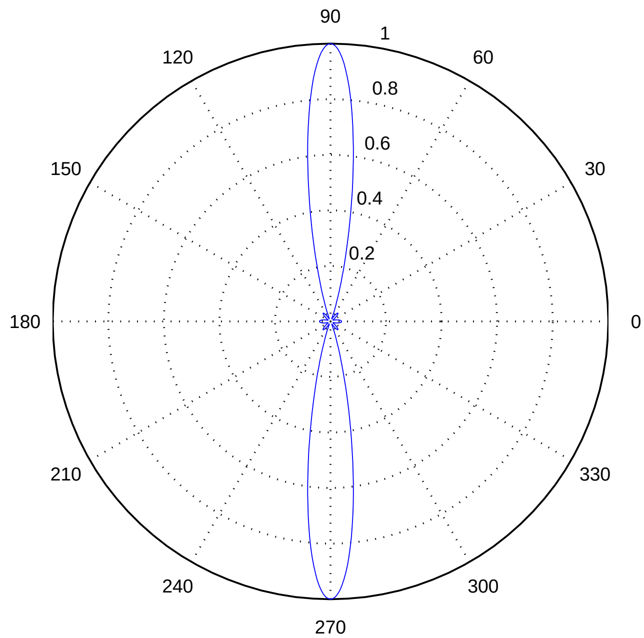


**Figure P9.47:** Normalized array pattern of a five-element array with uniform amplitude distribution in Problem 9.47.