

9.50 A three-element linear array of isotropic sources aligned along the z axis has an interelement spacing of $\lambda/4$ (Fig. P9.50). The amplitude excitation of the center element is twice that of the bottom and top elements, and the phases are $-\pi/2$ for the bottom element and $\pi/2$ for the top element, relative to that of the center element. Determine the array factor and plot it in the elevation plane.

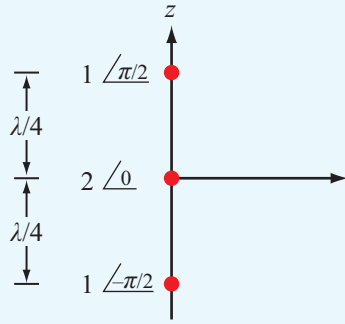


Figure P9.50 Three-element array of Problem 9.50.

Solution: From Eq. (9.110),

$$\begin{aligned}
 F_a(\theta) &= \left| \sum_{i=0}^2 a_i \exp j\psi_i \exp jikd \cos \theta \right|^2 \\
 &= |a_0 \exp j\psi_0 + a_1 \exp j\psi_1 \exp jkd \cos \theta + a_2 \exp j\psi_2 \exp j2kd \cos \theta|^2 \\
 &= |\exp j(\psi_1 - \pi/2) + 2 \exp j\psi_1 \exp j(2\pi/\lambda)(\lambda/4) \cos \theta \\
 &\quad + \exp j(\psi_1 + \pi/2) \exp j2(2\pi/\lambda)(\lambda/4) \cos \theta|^2 \\
 &= |\exp j\psi_1 \exp j(\pi/2) \cos \theta|^2 |\exp -j\pi/2 \exp -j(\pi/2) \cos \theta \\
 &\quad + 2 + \exp j\pi/2 \exp j(\pi/2) \cos \theta|^2 \\
 &= 4 \left(1 + \cos \left(\frac{1}{2} \pi (1 + \cos \theta) \right) \right)^2, \\
 F_{an}(\theta) &= \frac{1}{4} \left(1 + \cos \left(\frac{1}{2} \pi (1 + \cos \theta) \right) \right)^2.
 \end{aligned}$$

This normalized array factor is shown in Fig. 9.50(a).

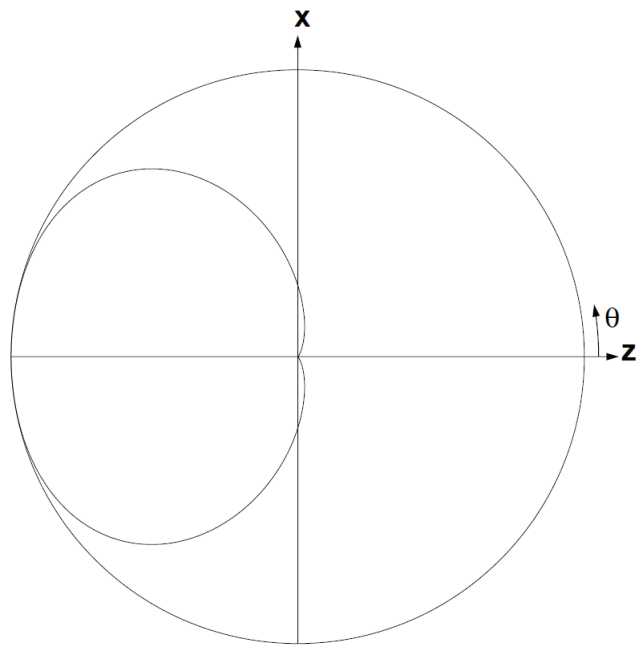


Figure P9.50(a): Normalized array pattern of the 3-element array of Problem 9.50.
