9.2 A 50 cm long center-fed dipole directed along the z direction and located at the origin is excited by a 1 MHz source. If the current amplitude is $I_0 = 10$ A, determine:

(a) The power density radiated at 2 km along the broadside of the antenna pattern.

(b) The fraction of the total power radiated within the sector between $\theta = 85^\circ$ and $\theta = 95^\circ$?

Solution:

(a)

$$\frac{l}{\lambda} = \frac{lf}{c} = \frac{0.5 \times 10^6}{3 \times 10^8} = 1.7 \times 10^{-3}.$$

Hence, the antenna qualifies as a Hertzian dipole with

$$S_0 = \frac{\eta_0 k^2 I_0^2 l^2}{32\pi^2 R^2} = \frac{\eta_0 (2\pi)^2 I_0^2 (l/\lambda)^2}{32\pi^2 R^2}$$
$$= \frac{377 \times 4 \times 100 \times (1.7 \times 10^{-3})^2}{32 \times 4 \times 10^6} = 3.27 \text{ (nW/m}^2\text{)}.$$

(b)

$$\text{Fraction} = \frac{\displaystyle\int_{\phi=0}^{2\pi} \int_{\theta=85^\circ}^{95^\circ} F(\theta, \phi) \sin\theta \, d\theta \, d\phi}{\displaystyle\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} F(\theta, \phi) \sin\theta \, d\theta \, d\phi}$$
$$= \frac{2\displaystyle\int_{85^\circ}^{90^\circ} \sin^3\theta \, d\theta}{2\displaystyle\int_{0}^{180^\circ} \sin^3\theta \, d\theta} = 6.52\%.$$