9.30 A uniformly illuminated aperture is of length $l_x = 20\lambda$. Determine the beamwidth between first nulls in the x–z plane.

Solution: The radiation intensity of a uniformly illuminated antenna is given by Eq. (9.90):

$$F(\theta) = \text{sinc}^2(\pi l_x \sin\theta/\lambda) = \text{sinc}^2(\pi\gamma),$$

with

$$\gamma = l_x \sin\theta/\lambda.$$

For $l_x = 20\lambda$,

$$\gamma = 20\sin\theta.$$

The first zero of the sinc function occurs when $\gamma = \pm 1$, as shown in Fig. 9-23. Hence,

$$1 = 20\sin\theta,$$

or

$$\theta = \sin^{-1}\left(\frac{1}{20}\right) = 2.87^\circ,$$

and

$$\beta_{\text{null}} = 2\theta = 5.73^\circ.$$