

Problem 1.13 The voltage of an electromagnetic wave traveling on a transmission line is given by $v(z, t) = 5e^{-\alpha z} \sin(4\pi \times 10^9 t - 20\pi z)$ (V), where z is the distance in meters from the generator.

- (a) Find the frequency, wavelength, and phase velocity of the wave.
- (b) At $z = 2$ m, the amplitude of the wave was measured to be 2 V. Find α .

Solution:

(a) This equation is similar to that of Eq. (1.28) with $\omega = 4\pi \times 10^9$ rad/s and $\beta = 20\pi$ rad/m. From Eq. (1.29a), $f = \omega/2\pi = 2 \times 10^9$ Hz = 2 GHz; from Eq. (1.29b), $\lambda = 2\pi/\beta = 0.1$ m. From Eq. (1.30),

$$u_p = \omega/\beta = 2 \times 10^8 \text{ m/s.}$$

- (b) Using just the amplitude of the wave,

$$2 = 5e^{-\alpha \cdot 2}, \quad \alpha = \frac{-1}{2 \text{ m}} \ln\left(\frac{2}{5}\right) = 0.46 \text{ Np/m.}$$
