

Problem 1.16 Evaluate each of the following complex numbers and express the result in rectangular form:

(a) $z_1 = 8e^{j\pi/3}$

(b) $z_2 = \sqrt{3} e^{j3\pi/4}$

(c) $z_3 = 2e^{-j\pi/2}$

(d) $z_4 = j^3$

(e) $z_5 = j^{-4}$

(f) $z_6 = (1 - j)^3$

(g) $z_7 = (1 - j)^{1/2}$

Solution: (Note: In the following solutions, numbers are expressed to only two decimal places, but the final answers are found using a calculator with 10 decimal places.)

(a) $z_1 = 8e^{j\pi/3} = 8(\cos \pi/3 + j \sin \pi/3) = 4.0 + j6.93.$

(b)

$$z_2 = \sqrt{3} e^{j3\pi/4} = \sqrt{3} \left[\cos \left(\frac{3\pi}{4} \right) + j \sin \left(\frac{3\pi}{4} \right) \right] = -1.22 + j1.22 = 1.22(-1 + j).$$

(c) $z_3 = 2e^{-j\pi/2} = 2[\cos(-\pi/2) + j \sin(-\pi/2)] = -j2.$

(d) $z_4 = j^3 = j \cdot j^2 = -j$, or

$$z_4 = j^3 = (e^{j\pi/2})^3 = e^{j3\pi/2} = \cos(3\pi/2) + j \sin(3\pi/2) = -j.$$

(e) $z_5 = j^{-4} = (e^{j\pi/2})^{-4} = e^{-j2\pi} = 1.$

(f)

$$\begin{aligned} z_6 &= (1 - j)^3 = (\sqrt{2} e^{-j\pi/4})^3 = (\sqrt{2})^3 e^{-j3\pi/4} \\ &= (\sqrt{2})^3 [\cos(3\pi/4) - j \sin(3\pi/4)] \\ &= -2 - j2 = -2(1 + j). \end{aligned}$$

(g)

$$\begin{aligned} z_7 &= (1 - j)^{1/2} = (\sqrt{2} e^{-j\pi/4})^{1/2} = \pm 2^{1/4} e^{-j\pi/8} = \pm 1.19(0.92 - j0.38) \\ &= \pm(1.10 - j0.45). \end{aligned}$$
