

Problem 1.18 Complex numbers z_1 and z_2 are given by

$$z_1 = -3 + j2$$

$$z_2 = 1 - j2$$

Determine (a) $z_1 z_2$, (b) z_1/z_2^* , (c) z_1^2 , and (d) $z_1 z_1^*$, all in polar form.

Solution:

(a) We first convert z_1 and z_2 to polar form:

$$\begin{aligned} z_1 &= -(3 - j2) = -\left(\sqrt{3^2 + 2^2} e^{-j \tan^{-1} 2/3}\right) \\ &= -\sqrt{13} e^{-j33.7^\circ} \\ &= \sqrt{13} e^{j(180^\circ - 33.7^\circ)} \\ &= \sqrt{13} e^{j146.3^\circ}. \end{aligned}$$

$$\begin{aligned} z_2 &= 1 - j2 = \sqrt{1+4} e^{-j \tan^{-1} 2} \\ &= \sqrt{5} e^{-j63.4^\circ}. \end{aligned}$$

$$\begin{aligned} z_1 z_2 &= \sqrt{13} e^{j146.3^\circ} \times \sqrt{5} e^{-j63.4^\circ} \\ &= \sqrt{65} e^{j82.9^\circ}. \end{aligned}$$

(b)

$$\frac{z_1}{z_2^*} = \frac{\sqrt{13} e^{j146.3^\circ}}{\sqrt{5} e^{j63.4^\circ}} = \sqrt{\frac{13}{5}} e^{j82.9^\circ}.$$

(c)

$$\begin{aligned} z_1^2 &= (\sqrt{13})^2 (e^{j146.3^\circ})^2 = 13 e^{j292.6^\circ} \\ &= 13 e^{-j360^\circ} e^{j292.6^\circ} \\ &= 13 e^{-j67.4^\circ}. \end{aligned}$$

(d)

$$\begin{aligned} z_1 z_1^* &= \sqrt{13} e^{j146.3^\circ} \times \sqrt{13} e^{-j146.3^\circ} \\ &= 13. \end{aligned}$$
