

Problem 1.19 If $z = -2 + j4$, determine the following quantities in polar form:

- (a) $1/z$,
- (b) z^3 ,
- (c) $|z|^2$,
- (d) $\Im\{z\}$,
- (e) $\Im\{z^*\}$.

Solution: (Note: In the following solutions, numbers are expressed to only two decimal places, but the final answers are found using a calculator with 10 decimal places.)

(a)

$$\frac{1}{z} = \frac{1}{-2 + j4} = (-2 + j4)^{-1} = (4.47 e^{j116.6^\circ})^{-1} = (4.47)^{-1} e^{-j116.6^\circ} = 0.22 e^{-j116.6^\circ}.$$

$$(b) \ z^3 = (-2 + j4)^3 = (4.47 e^{j116.6^\circ})^3 = (4.47)^3 e^{j350.0^\circ} = 89.44 e^{-j10^\circ}.$$

$$(c) \ |z|^2 = z \cdot z^* = (-2 + j4)(-2 - j4) = 4 + 16 = 20.$$

$$(d) \ \Im\{z\} = \Im\{-2 + j4\} = 4.$$

$$(e) \ \Im\{z^*\} = \Im\{-2 - j4\} = -4 = 4e^{j\pi}.$$
